Dynamical properties of the Solar System

Suggested reading:

Physics of the Earth and the Solar System

B. Bertotti and P. Farinella, Kluwer Academic Publishers

Planets and Astrobiology (2020-2021)

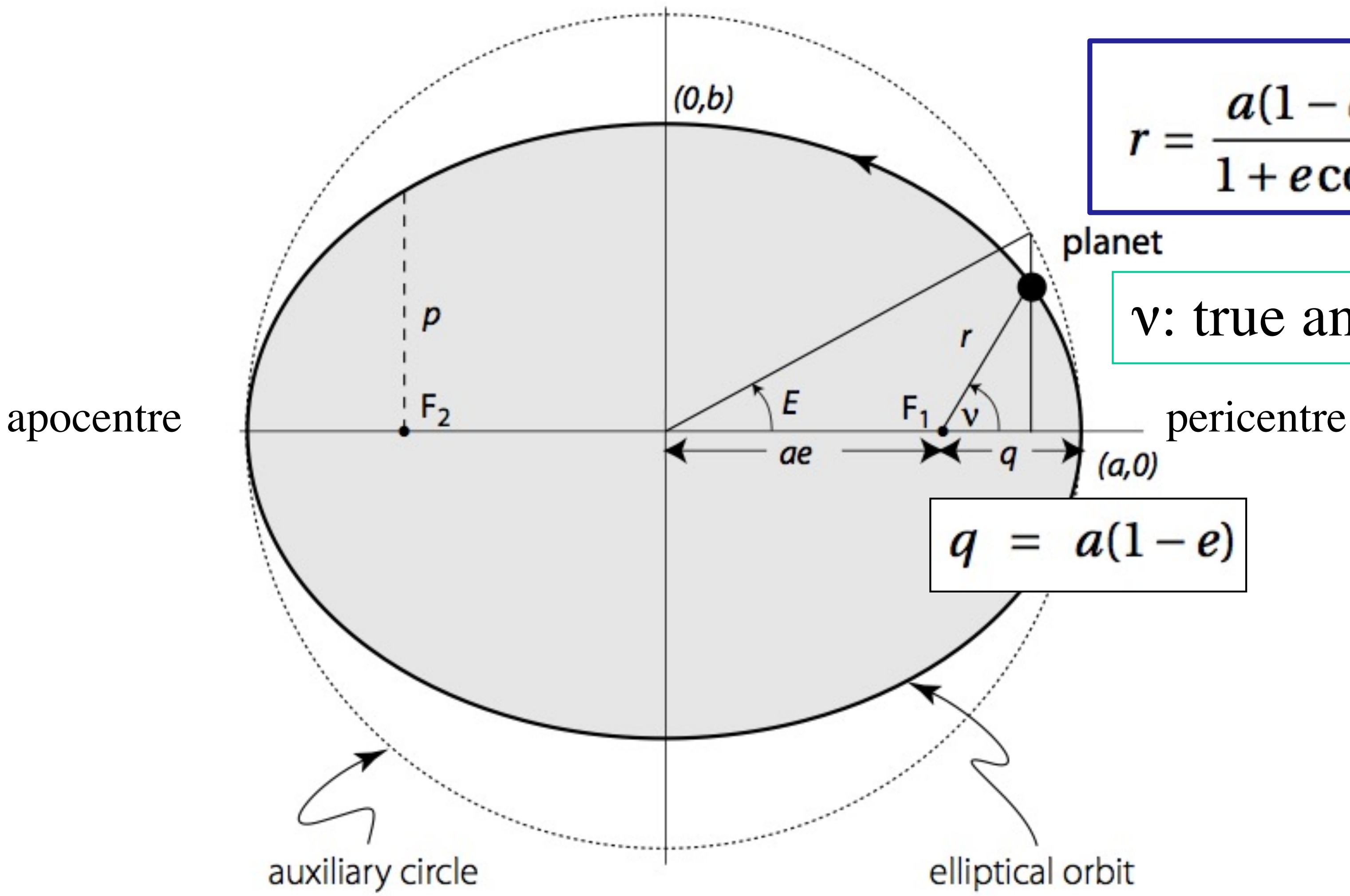
G. Vladilo

Dynamics of planetary orbits

- In the limit of non-relativistic motion, Newton's laws can be applied
  - Relativistic effects should be taken into account for orbits very close to the star
- Even using the newtonian approximation, the treatment of N-body systems, such as the Solar System, is extremely complex
- However, over a short period of time, the motion of a planet around its host star can be treated as a 2-body problem, neglecting perturbations due to other bodies in the system
- In this limit, two objects can be considered to be isolated in space and to move around each other according to Keplerian laws
  - Kepler's laws result from the inverse-square law of gravity, with the Sun as the central body, and the conservation of angular momentum and energy
  - These laws are fundamental to understand the dynamics of the Solar System and extrasolar planetary systems

The first Kepler's Law and its physical meaning

The secondary body moves in an <u>elliptical orbit</u>, with the primary body at the focus



$$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

ν: true anomaly

$$q = a(1-e)$$

The conservation principle behind the first Kepler's Law

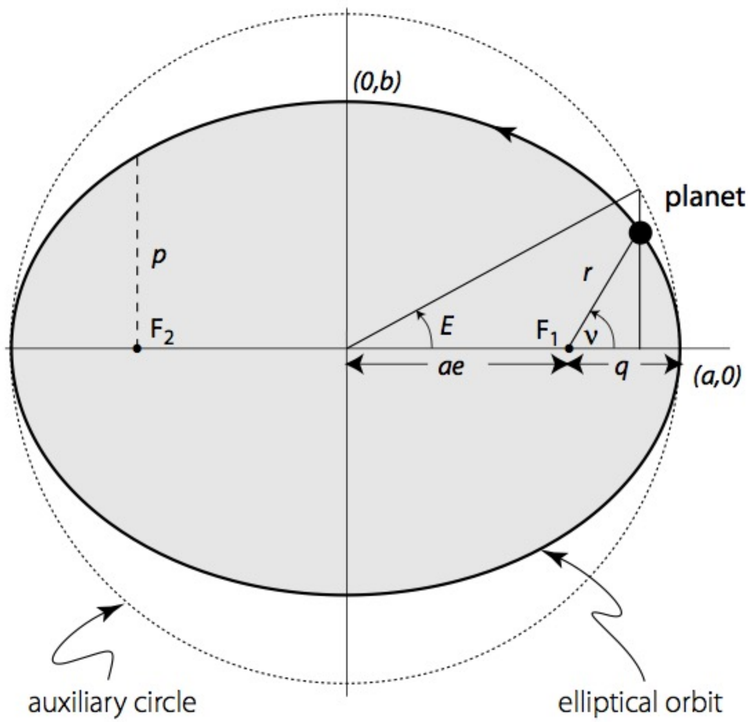
Valid for bound orbits with $E < 0$

Semi-major axis

$$a = -G\,(m_1+m_2)/(2E)$$

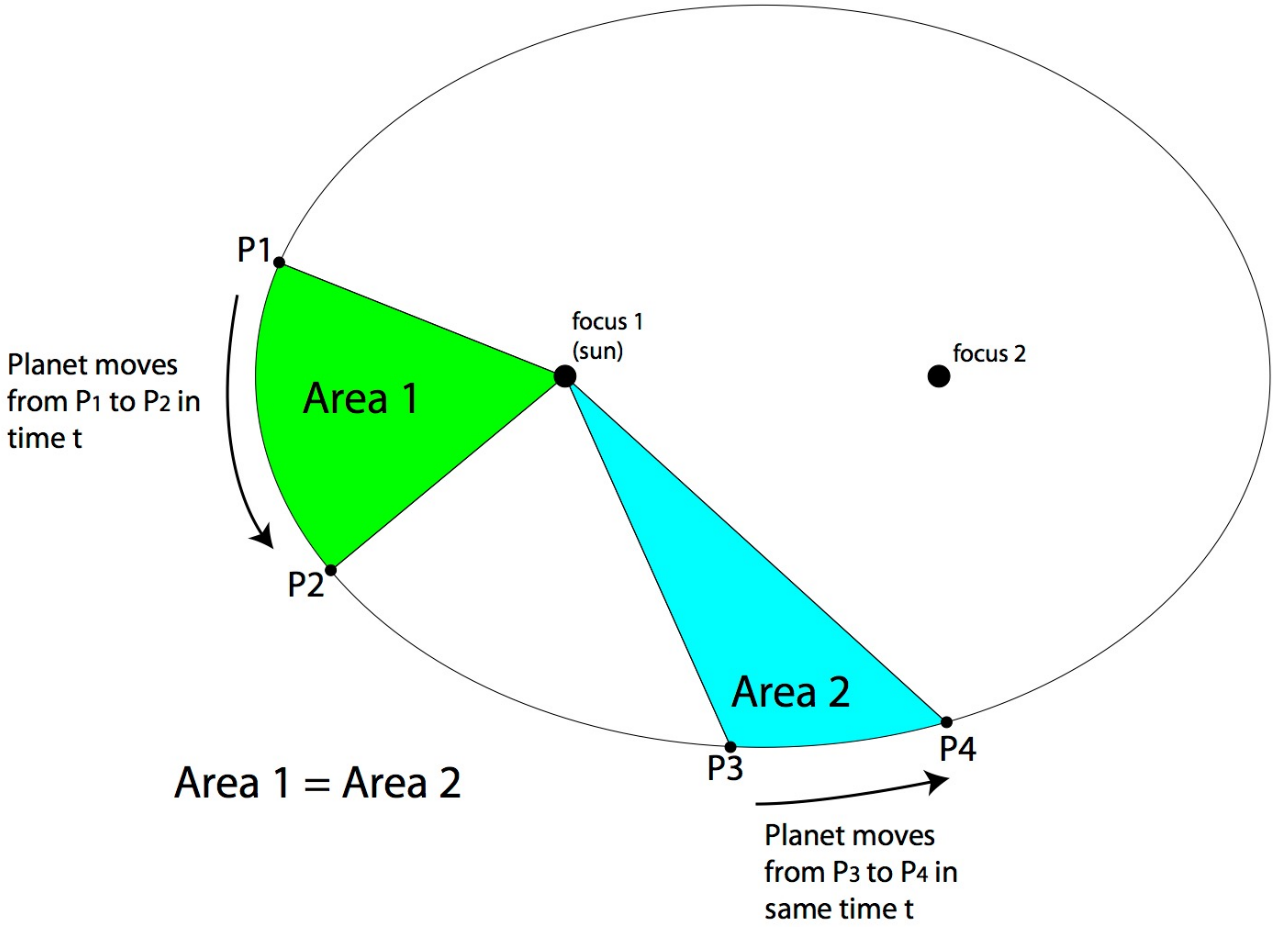
The conservation of the total energy *E*

yields a constant semi-major axis



Second Kepler's Law

The straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time



Physical meaning of the second Kepler's Law

$$dA = \frac{1}{2}r^2 d\theta$$

dA: infinitesimal area swept by an infinitesimal displacement $d\theta$ of the radial vector *r*

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

Area swept per unit time, where ω is the angular velocity

$$L = mr^2\omega$$

L: angular momentum

$$\frac{dA}{dt} = \frac{L}{2m}$$

Kepler's 2^{nd} Law follows from the conservation of the angular momentum

Third Kepler's Law

Original formulation, based on empirical evidence: The square of a planet's orbital period P about the Sun (in years) equals the cube of its semimajor axis a (in AU)

$$P^2 = a^3$$

By solving the two-body problem one can derive the more general expression

$$P^2 = \frac{4\,\pi^2\,a^3}{G\,(M_* + M_{\rm p})}$$

where M_* and M_p are the mass of the central star and planet, respectively.
In general $M_p << M_*$, so in practice we have

$$P^2 \simeq \frac{4\pi^2 a^3}{GM_*}$$

Third Kepler's Law
Physical interpretation for circular orbits

The third Kepler's law for $M_p << M_*$ can be easily derived for circular orbits.

By equating the centripetal force of the planet with the gravitational force exerted by the central body, we have

$$\frac{M_p v_c^2}{a} = G\frac{M_p M_*}{a^2}$$

where a is the orbital radius.

From this we obtain the velocity

$$v_c = \sqrt{\frac{GM_*}{a}}$$

and the period

$$P = \frac{2\pi a}{v_c} = 2\pi\sqrt{\frac{a^3}{GM_*}}$$

This last expression yields the third Kepler's law for the case $M_p << M_*$

Orbital elements

- In order to locate the positions of a body moving in an elliptical orbit in 3D space, a set of 6 orbital parameters (or elements) is required
- The orbital elements specify the size, shape, and orientation of the orbit in space and the position of the body at a particular instant in time (the “epoch”)

- Commonly used parameters are

  a = the semi-major axis of the ellipse

  e = the eccentricity of the ellipse

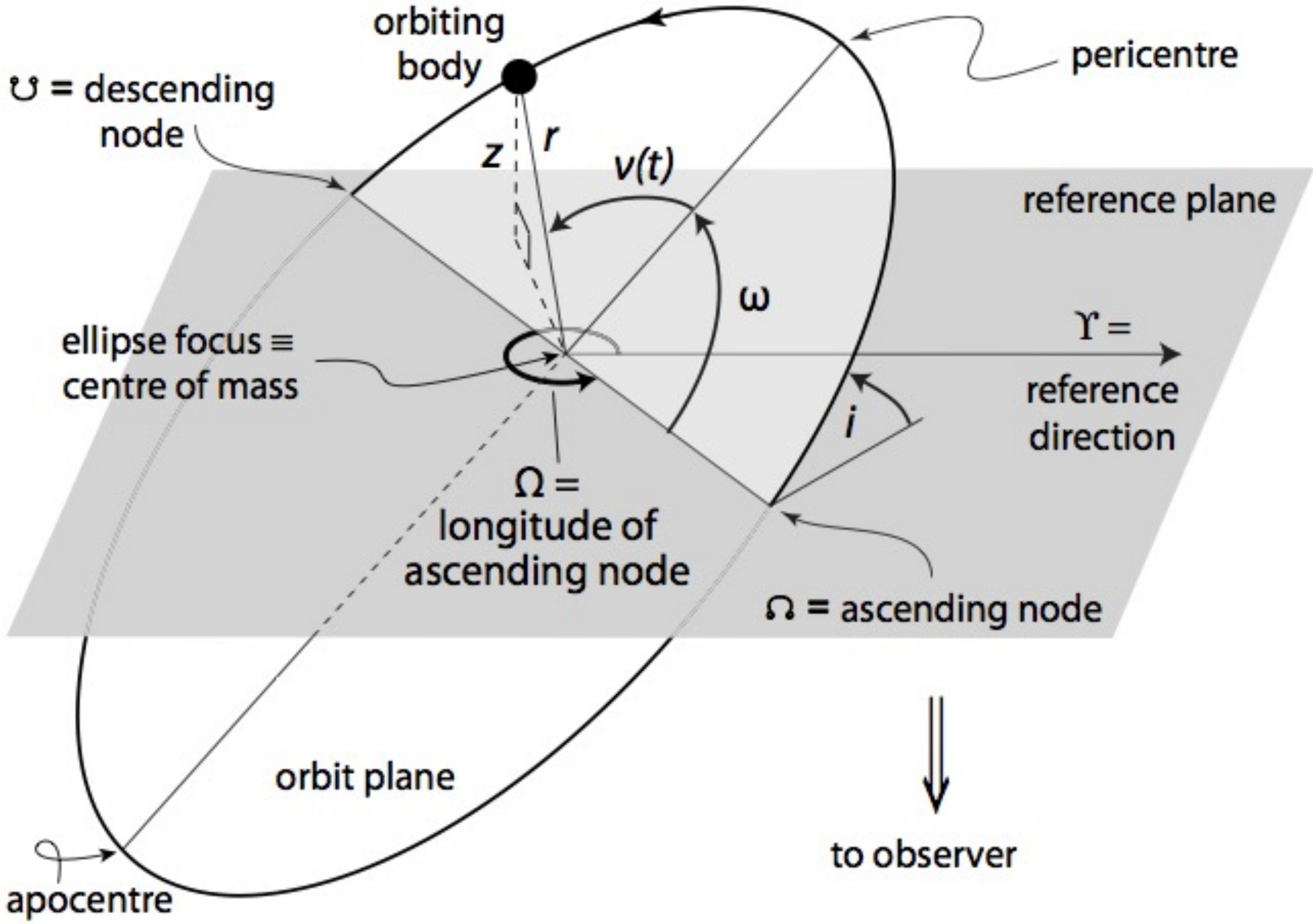
  i = the inclination of the orbital plane to the reference plane

  Ω = the longitude of the node of the orbital plane on the reference plane

  ω = the argument of the pericenter

  T = the epoch at which the body is at the pericenter

Orbital elements



Orbital parameters of the Solar System planets

Object	a	P	e	I
Sun	-	-	-	-
Mercury	0.387	87.97d	0.206	7.00
Venus	0.723	224.70d	0.007	3.39
Earth	1.000	365.25d	0.017	0.00
Mars	1.524	686.98d	0.093	1.85
Jupiter	5.203	11.86y	0.048	1.30
Saturn	9.539	29.46y	0.056	2.49
Uranus	19.182	84.01y	0.047	0.77
Neptune	30.058	164.79y	0.009	1.77
Pluto	39.44	247.7y	0.250	17.2

Table 14.1. Dynamical properties of the planets and the sun: a is the semimajor axis in AU; P is the orbital period; e the eccentricity; I the inclination in degrees; P_s the spin period in days; ϵ the obliquity to the ecliptic in degrees.

- Period and semimajor axis obey to 3rd Kepler's law
- Eccentricity is generally low
- Orbits are approximately coplanar, with small inclinations *I* relative to the ecliptic
- Orbital spins are generally aligned with the spin of solar rotation

Rotation periods and tilt of rotation axis

Object	a	P	e	I	P_s	ϵ
Sun	-	-	-	-	25.4	7.25
Mercury	0.387	87.97d	0.206	7.00	58.65	0
Venus	0.723	224.70d	0.007	3.39	243.0	178
Earth	1.000	365.25d	0.017	0.00	1.00	23.4
Mars	1.524	686.98d	0.093	1.85	1.026	25.0
Jupiter	5.203	11.86y	0.048	1.30	0.410	3.08
Saturn	9.539	29.46y	0.056	2.49	0.426	26.7
Uranus	19.182	84.01y	0.047	0.77	0.720	97.9
Neptune	30.058	164.79y	0.009	1.77	0.670	28.8
Pluto	39.44	247.7y	0.250	17.2	6.387	94

Table 14.1. Dynamical properties of the planets and the sun: a is the semimajor axis in AU; P is the orbital period; e the eccentricity; I the inclination in degrees; P_s the spin period in days; ϵ the obliquity to the ecliptic in degrees.

Rotation is generally prograde with the rotation spin of the Sun
Obliquity with respect to the ecliptic generally < 30 degrees
Exceptions: Venus, Uranus

Gravitational perturbations

- When more than 2 bodies are present, the gravitational perturbations induced by the other bodies will alter the orbital and rotational parameters of the system
  - Example: precession of the pericenter which, in the case of the Earth, yields the well-known precession of the equinox

- Different types of perturbations exist, characterized by different intensities and time scales
  - To study perturbations we introduce the concept of resonance

Resonances

- When the ratio of dynamical periodicities of two bodies can be expressed as the ratio of two small integers, gravitational perturbations tend to cumulate, leading to a resonance

$$P_1/P_2 = n/m$$

- Resonances may stabilize or destabilize orbits

- Since the orbital period P and semimajor axis a are linked though the 3rd Kepler's law, <u>resonances play a key role in shaping the architecture of planetary systems</u>

Resonances

- Orbital resonance

$$P_{1,\mathrm{orb}}/P_{2,\mathrm{orb}} = n/m$$

  - When the orbital periods are in resonance
  - It is called "MEAN MOTION ORBITAL RESONANCE"
  - Orbital resonance can generate a dynamical instability
  - In the most extreme case, it may lead to the ejection of one of the two bodies from the system in dynamical (i.e., fast) time scales

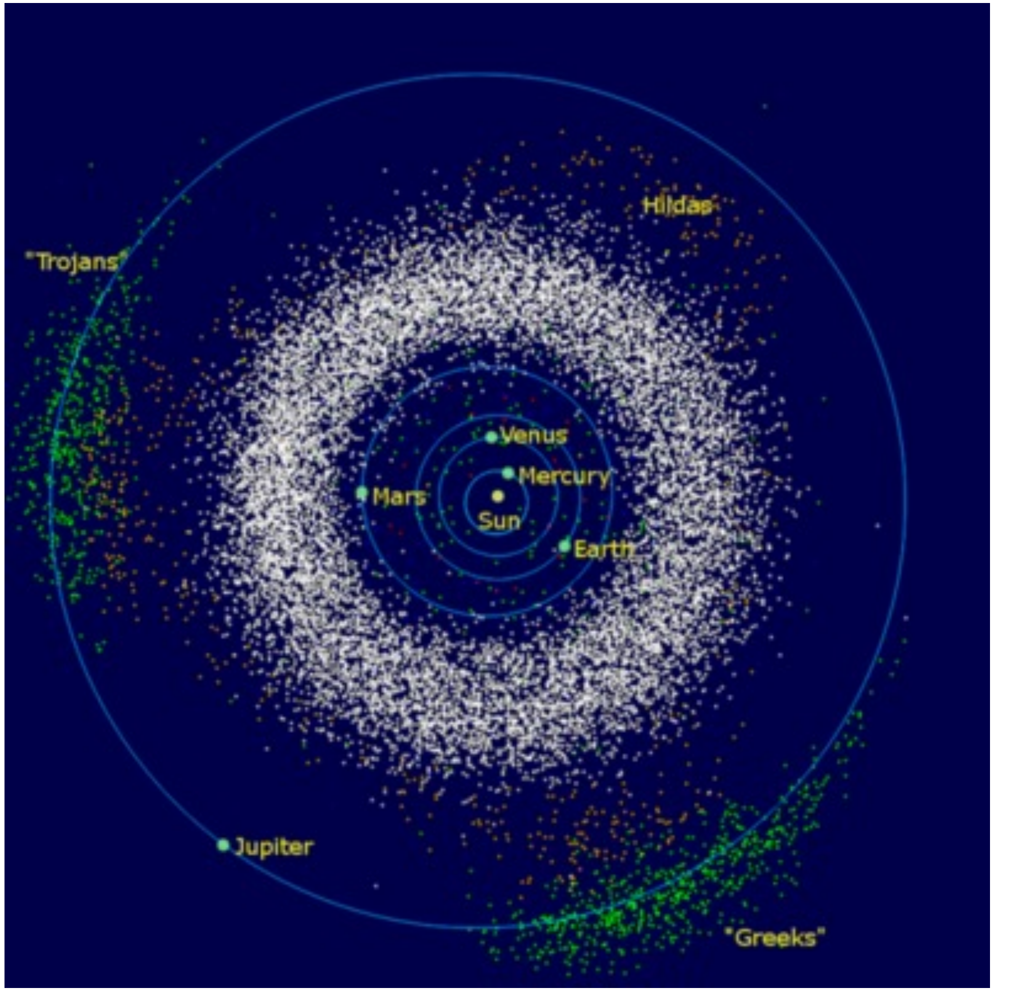
- Secular resonance

$$P_{1,\mathrm{prec}}/P_{2,\mathrm{prec}} = n/m$$

  - When the periods of precession of the periapsides are in resonance
  - Secular resonances may lead to gradual changes of the orbital eccentricity and inclination on longer time scales, of the order of millon years

Orbital resonances
Examples of orbital stabilization

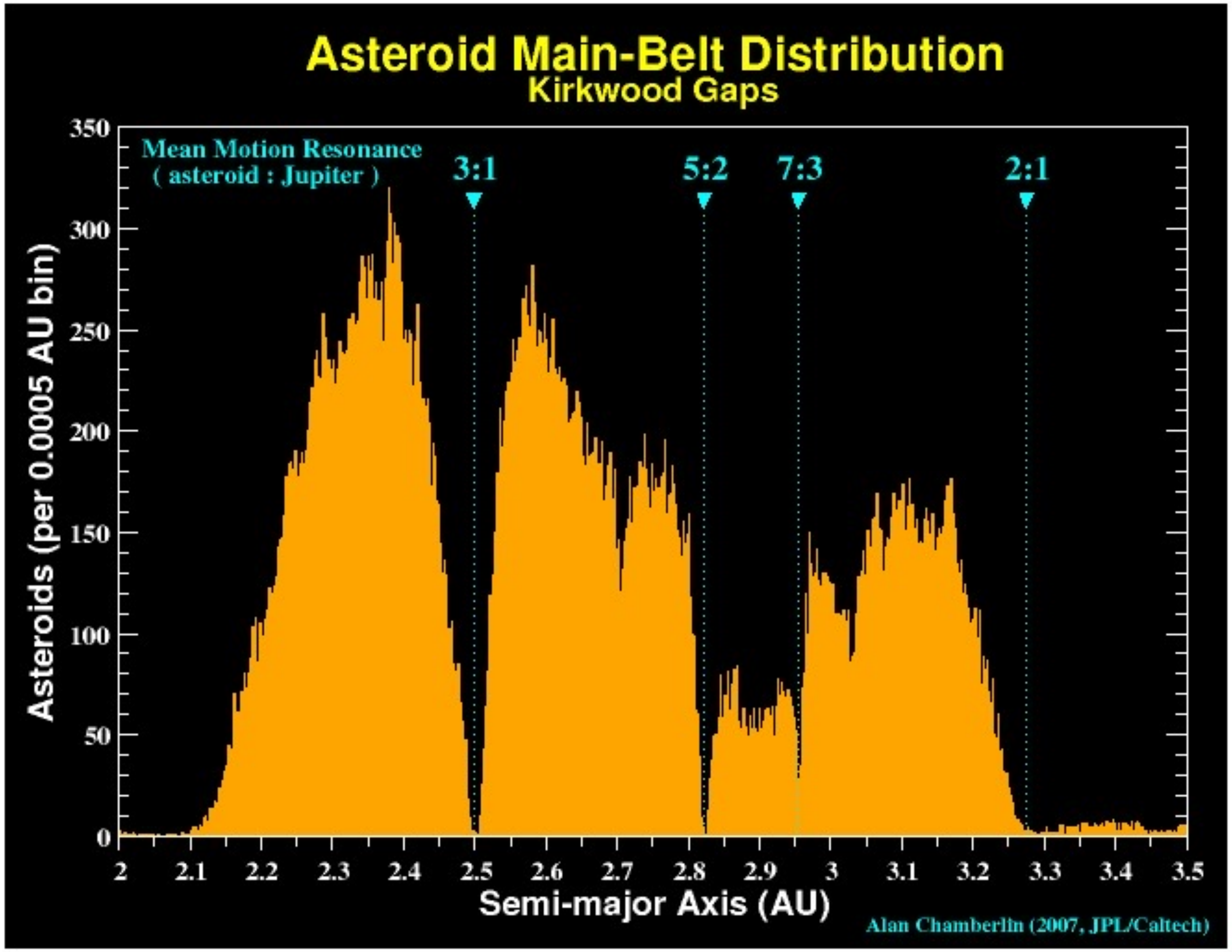
- Concentrations of minor bodies stabilized by dynamical effects
  - Hilda family: orbital resonance 3:2 with Jupiter
  - Trojans: resonance 1:1 with Jupiter



Orbital resonances
Examples of orbital destabilization

Distribution of semimajor axis in the main asteroid belt

Gaps result from the ejection of asteroids from regions in resonance with Jupiter's orbital period

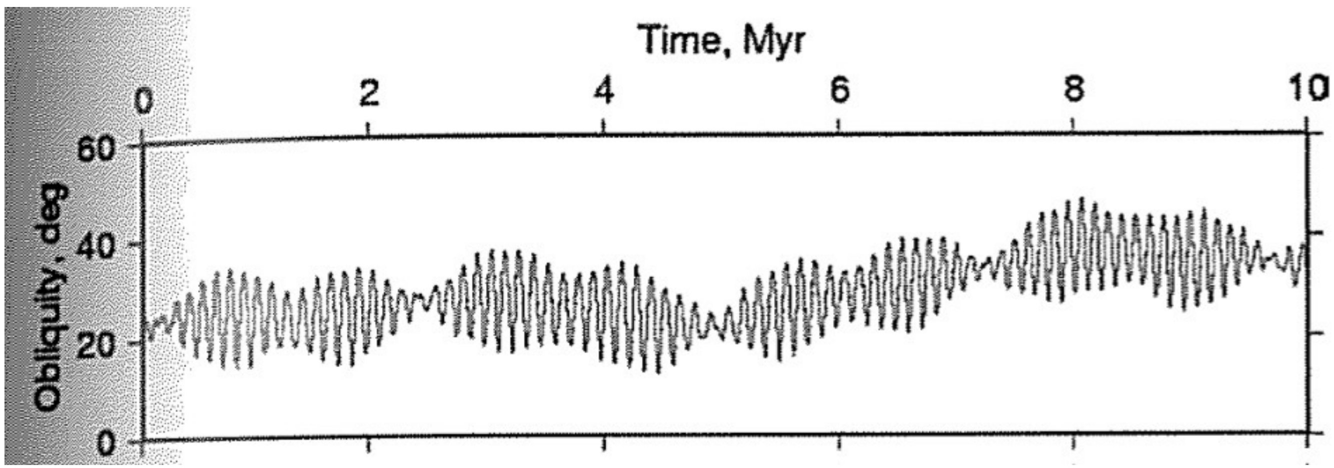


Other examples of resonances

– The architecture of rings around giant planets results from resonance effects between debris material that surrounds the planets and satellites orbiting the same planets
– Such resonance effects generate gaps in the debris material, leading to the formation of the characteristic ring pattern

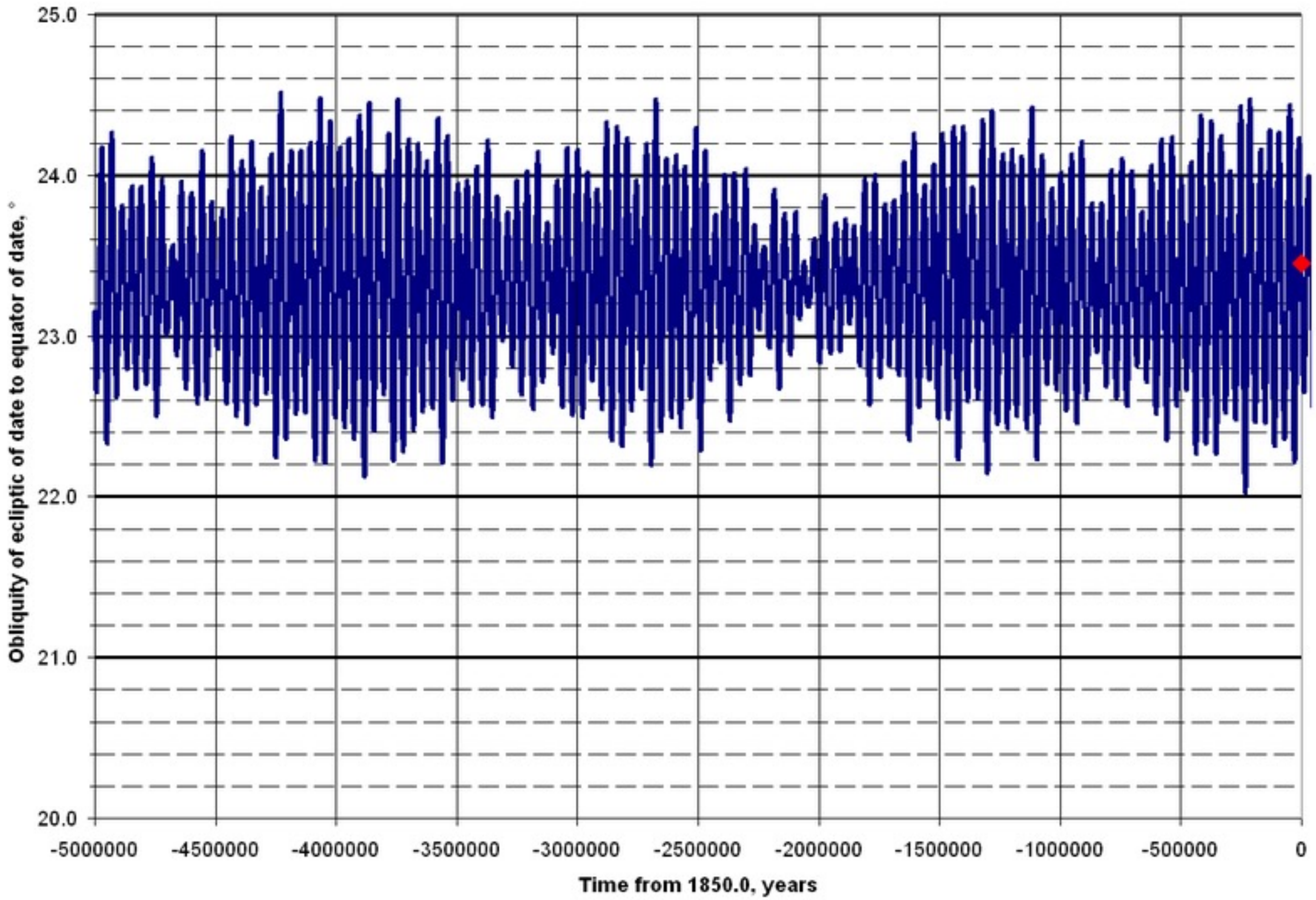
Effects of gravitational perturbations on the tilt of the planetary rotation axis

- The tilt of the rotation axis influences the planetary climate and habitability
- Gravitational torques between different planets tend to alter the tilt of the rotation axis in a chaotic fashion
  - For instance, the axis tilt of Mars is believed to evolve significantly, leading to extreme inclinations in some epochs



At variance with Mars, the axist tilt of the Earth is stabilized by the presence of the Moon

Therefore, the Earth's axis experiences small fluctuations of obliquity around its typical value of 23.5 degrees



Tidal effects

Tidal effects are ubiquitous in the universe

When two bodies interact gravitationally and the size of at least one of them is not negligible with respect to their mutual distance

$$R \sim r$$

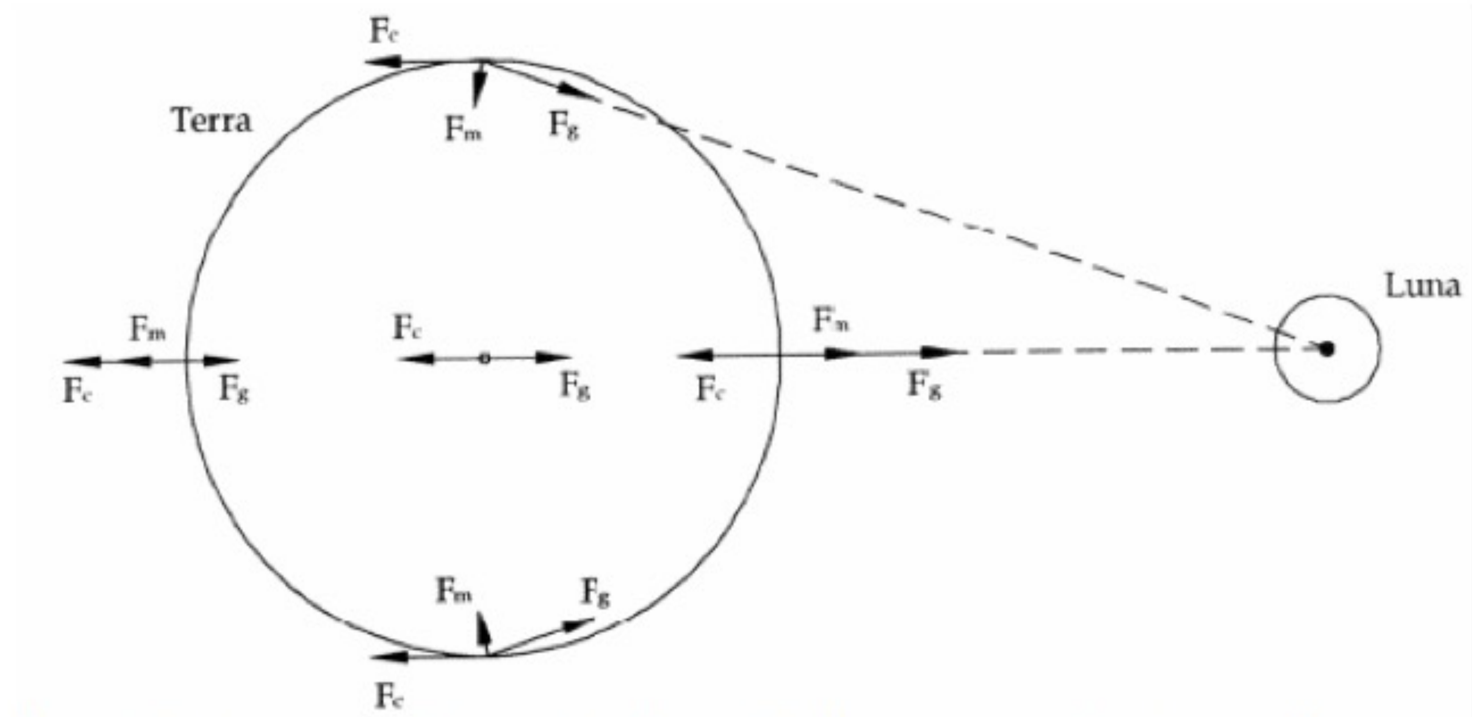
In this case gravity gradients give rise to forces and torques that cause <u>deformations</u> and <u>changes in the rotational state</u> of affected bodies

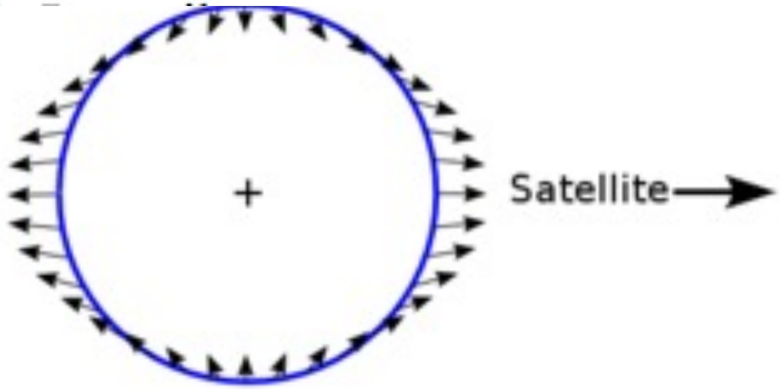
Tidal forces tend to vanish very rapidly with distance

From differentiation of Newton's law in distance, r

$$f_{\mathrm{T}} \sim r^{-3}$$

Surface tides





Surface tides can be estimated by calculating the vectorial sum of the centrifugal force and the gravitational force (with its gradient inside the planet or satellite)

Tidal locking

- Mechanism
  - Tidal forces create a bulge
  - If $P_{rot} < P_{orb}$, the body rotates through its tidal bulge, generating internal friction
  - Friction slows downs the rotation (P_{rot} increases)
  - The friction stops when $P_{rot} = P_{orb}$
- We say that a body is “tidally locked” when tidal effects have slowed the rotation period down to $P_{rot} = P_{orb}$

- Examples
  - Satellites around planets: Moon-Earth and Titan-Saturn systems
  - Important for extrasolar planets close to their central star

Long-term dynamical stability of the Solar System

- How stable is the Solar System ?
  - We are not able to provide a simple analytical description of the dynamical evolution of a set of N > 3 bodies under the effects of recyprocal gravitational interaction
  - Assessing the long-term dynamical stability of the Solar System requires N-body gravitational simulations
- N-body simulations indicate that small variations of the orbital parameters of the solar system bodies lead to unstable configurations, with possible ejection of a body from the system
- This type of investigation confirm that dynamical interactions are essential in shaping a stable architecture of planetary systems