

Models of Planetary System formation

Planets and Astrobiology (2023)

G. Vladilo and S. Ivanovski

Models of planetary formation

(Armitage 2009, Lecture notes on the formation and early evolution of planetary systems)

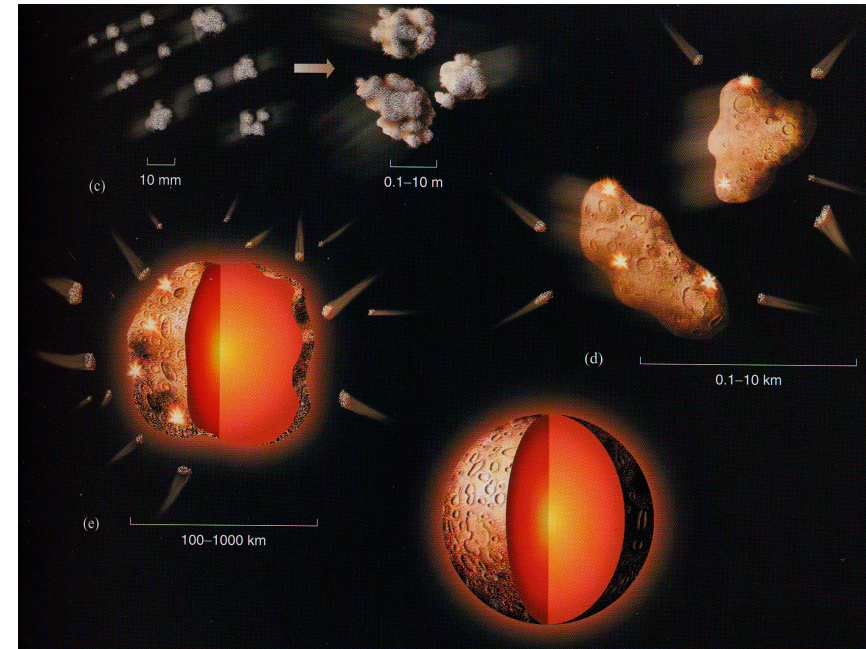
Models of planetary formation were originally aimed at reconstructing the history of the Solar System formation

Currently, they aim at interpreting the observations of protoplanetary disks and exoplanetary systems

Historically, different types of models have been developed for terrestrial planets and giant planets

Models of formation of terrestrial planets

- **The planetesimal hypothesis**
 - The solid component of the protoplanetary disk undergoes several steps of accretion until bodies with sizes in the order of a few kilometers, called planetesimals, are formed
 - Collisions and gravitational interactions between planetesimals originate planetary embryos, with sizes in the interval between the Moon and Mars
 - The embryos accrete planetesimals and collide until they form terrestrial planets



Models of formation of terrestrial planets

During the process of terrestrial planet formation the solid component undergoes the following changes of the mean size, a

Dust grains

$$0.1 \mu\text{m} \leq a \leq 1 \text{ cm}$$

Rock/boulders

$$a \sim 1 \text{ m}$$

Planetesimals

$$a \sim 10 \text{ km}$$

Planetary embryos

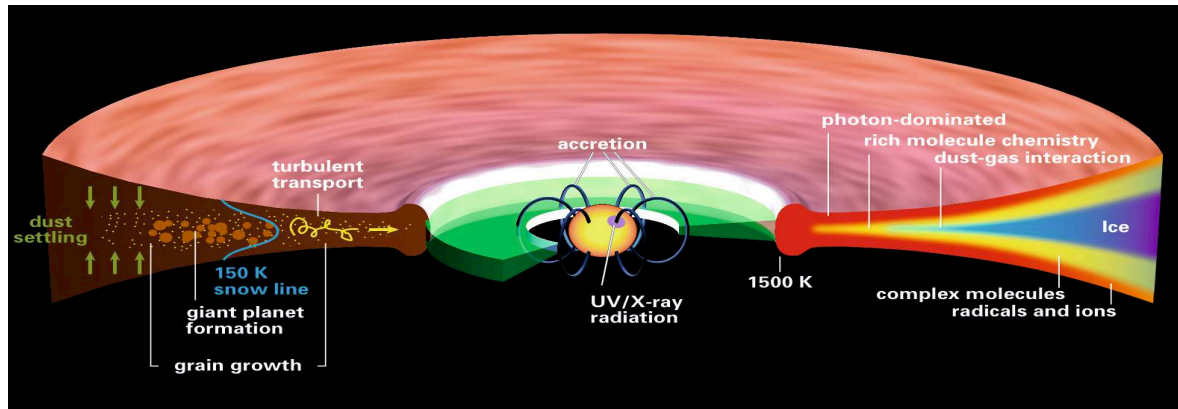
$$a \sim R_{\text{Moon}}$$

Planets

$$a \sim R_{\text{Earth}}$$

Therefore the process of planetary formation requires an accretion that extends over more than 12 orders of magnitude in size

Structure, grain evolution processes and observational constraints for protoplanetary disks



Testi et al.2014

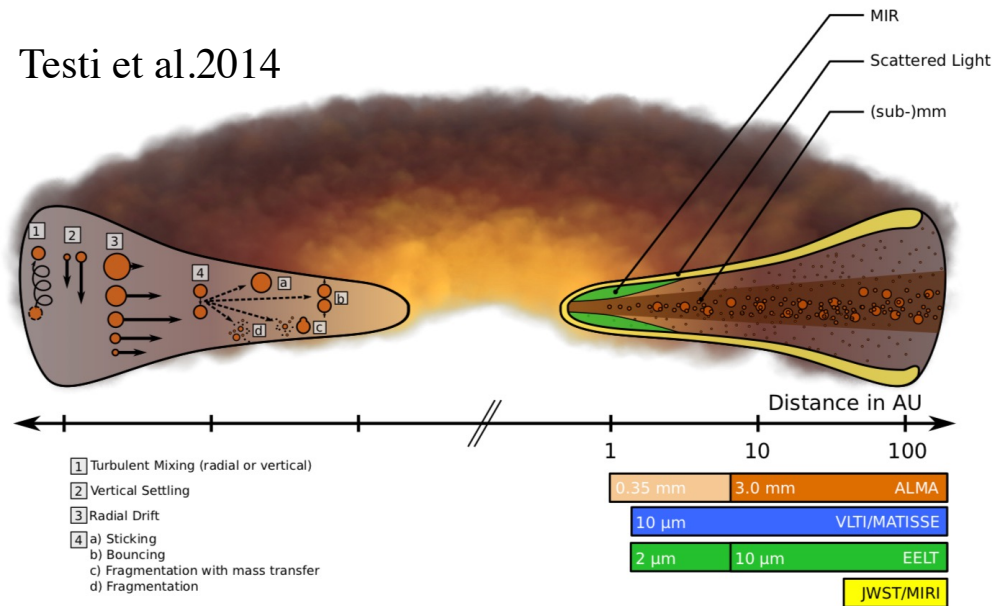


Fig. 1.— Illustration of the structure, grain evolution processes and observational constraints for protoplanetary disks. On the left side we show the main grain transport and collision mechanism properties. The different lengths of the arrows illustrate the different velocities of the different grains. On the right hand side, we show the areas of the disk that can be probed by the various techniques. The axis shows the logarithmic radial distance from the central star. The horizontal bars show the highest angular resolutions (left edge of the bars) that can be achieved with a set of upcoming facilities and instruments for at the typical distance of the nearest star forming regions.

Models of formation of terrestrial planets

- The physical mechanisms that govern the accretion of the solid component vary according to the value of a
 - Interactions between the solid and gaseous component dominate at low values of a
 - Gravitational interactions dominate at high values of a
- With varying a the physical regimes become so different that each stage must be treated with a different type of modelization
 - In reality, the different processes may overlap in time and space
- Given the complexity of the models, the effects of the strong protostellar activity are often ignored in the models
 - To first approximation, the jets do not interfere with the protoplanetary disk
 - The protostellar emission, also in high-energy spectral bands, and magnetic fields are important

From dust to boulders

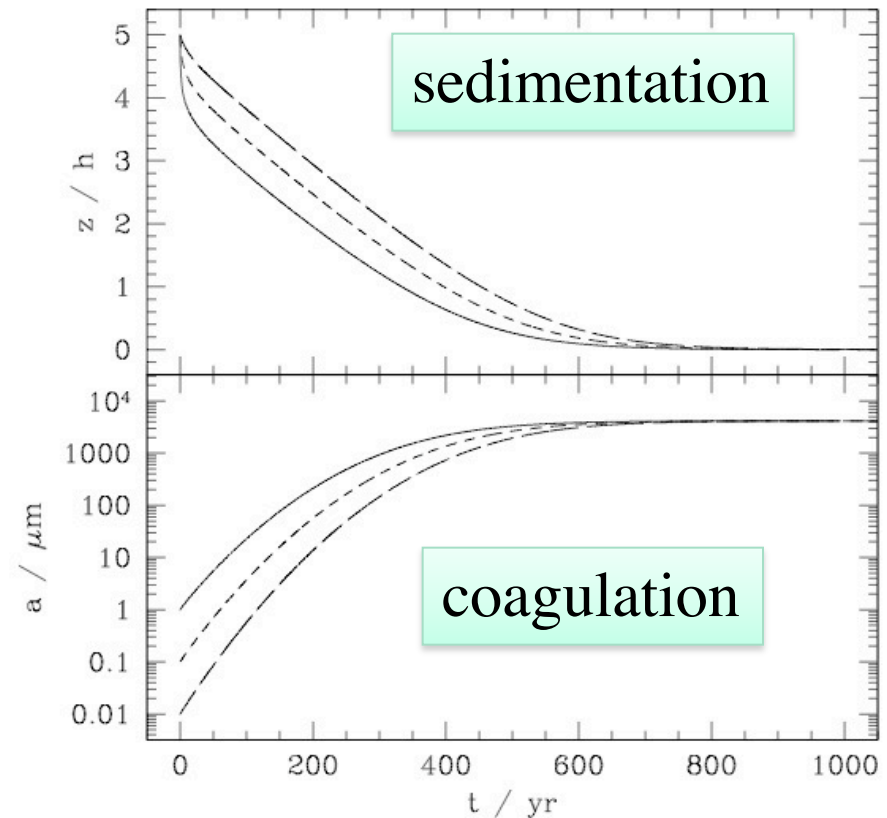
- **Size regime:**

$$a \sim 0.1 \mu\text{m} \Rightarrow a \sim 1 \text{ m}$$

- Gravitational forces between particles are completely negligible
- The growth of solid particles takes place via collision and coagulation (agglomeration)

The coalescence of the particles is determined by electrostatic forces

- The process of coagulation is concomitant with the vertical settling (sedimentation) of the dust grains onto the central plane of the protoplanetary disk
- The time scales for these initial stages are expected to be quite short (see figure)



Streaming instability and pebble accretion

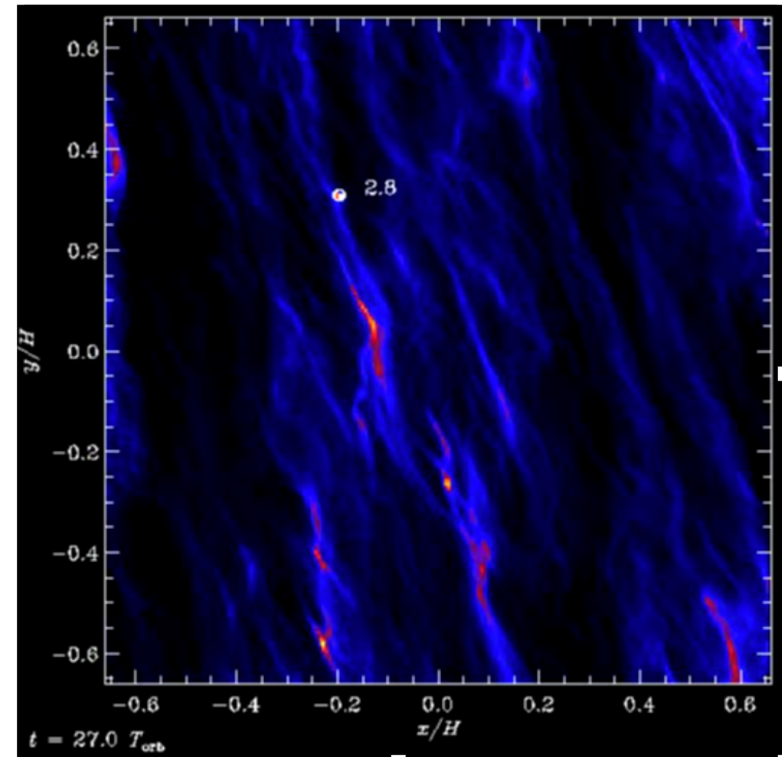
- **Size regime:**

$a \sim 1 \text{ mm} \Rightarrow a \sim 10 \text{ m}$

- Coagulation no longer efficient
- Collisions are too fast and net result is destruction, not growth
- Typical collision velocity, 100 m/sec

Meter-size barrier

- **Streaming instability** can help to overcome the meter-size barrier: particles are locally concentrated along the Keplerian flow direction. This can lead to formation of larger bodies



Lambrechts & Johansen 2012

From boulders to planetesimals

- Size regime:

$$a \sim 1 \text{ m} \Rightarrow a \sim 10 \text{ km}$$

- Gravitational forces between particles are still non dominant
- Not clear if at this stage the growth takes place by agglomeration or through other forms of local accumulation of solids
- Processes alternative to agglomeration are invoked since this step is quite critical due to radial drift that is expected to take place in the early stages of solid condensation

Early stages of solid condensation

- Radial drift

- Solid particles tend to move at Keplerian velocity, V_k

Determined by the equilibrium between gravitational and centrifugal forces

$$V_k^2/r = GM_*/r^2$$

- As a result of the pressure gradient inside the nebula, gas particles tend to move at velocities slightly lower than the Keplerian velocity

From the condition of hydrostatic equilibrium, one obtains:

$$V_g^2/r = GM_*/r^2 + (1/\rho) dP/dr$$

Since the pressure decreases with increasing distance from the star, $dP/dr < 0$ and therefore $V_g < V_k$

- The differential velocity between the solids, moving at Keplerian velocity, and the gas, moving slower, generates a friction
- Solid particles feel a “head wind”, lose angular momentum and undergo a “radial drift” towards the inner part of the disk

From boulders to planetesimals: the radial drift problem

- **Size dependence of the radial drift**
 - The radial drift becomes more intense with increasing particle size
The effect is predicted to be maximum at $a \sim 10 \text{ cm} - 1 \text{ m}$
“Rocks” with these sizes feel a strong “headwind” that forces them to spiral towards the star on a very short time scale (10^2 yr !)
- **Implications of the rapid radial drift of rocks**
 - The short time scale of the radial drift of rocks would abruptly interrupt the process of solid condensation (and planetary formation)
 - A mechanism must be invoked that allows a fast accumulation of rocks up to planetesimal size in order not to interrupt the process of planetary formation
 - Once planetesimals are formed, the radial drift becomes ineffective

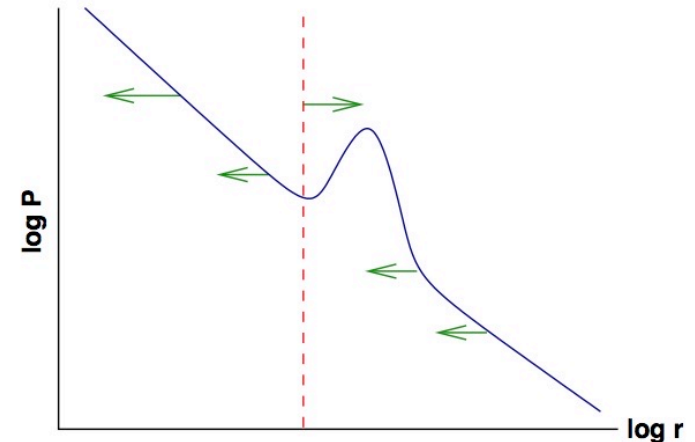
From boulders to planetesimals: mechanisms that may prevent the fast radial drift of boulders

- A local maximum in the radial pressure distribution would drive a local accumulation of solid material

The radial drift would move the rocks towards the maximum of pressure

This would allow accumulation of material present in the region of the disk where the pressure has a local maximum

Current models of planetary formation tend to invoke this type of mechanism

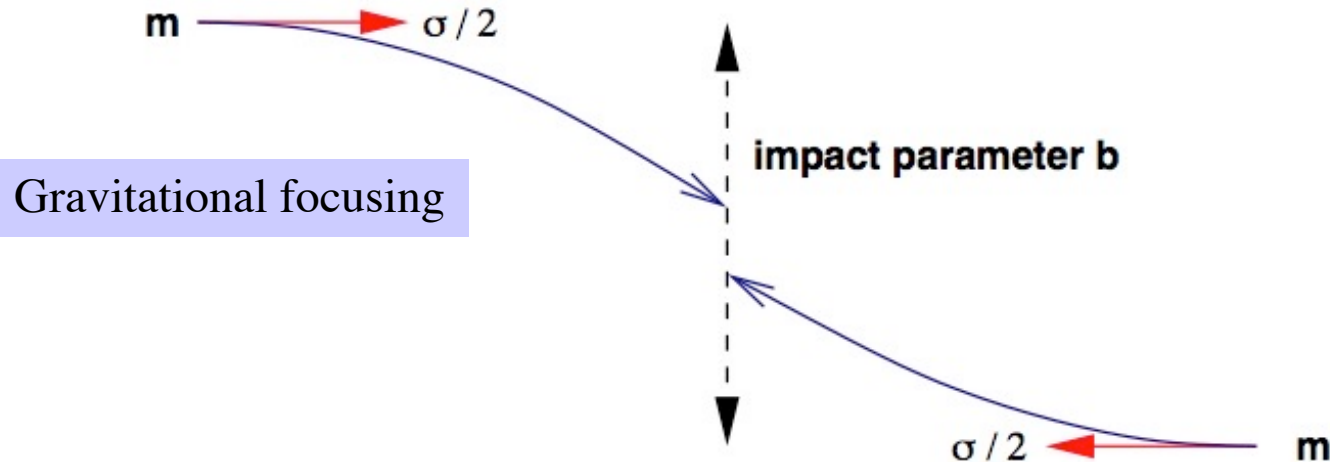


From planetesimals to planetary embryos

- Once planetesimals are formed, gravitational perturbations start to become important
 - The coupling with the gas becomes negligible (until the size becomes much larger)
 - In principle, this stage can be treated as an N -body problem dominated by gravitational interactions
 - In practice, even the numerical solution of this problem is difficult given the large number of particles involved ($N \sim 10^9$)
 - Growth of planetesimals takes place through collisions, provided the relative velocities are sufficiently low to avoid disruption

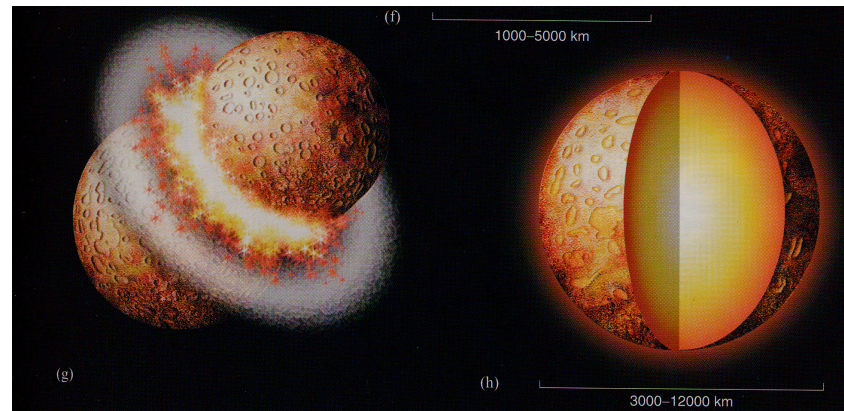
From planetesimals to planetary embryos

- As the masses become larger, gravitational focusing becomes important
- A phase of runaway growth occurs in which a few bodies grow rapidly at the expense of the rest
- Runaway growth ceases once the largest bodies become massive enough to stir up the planetesimals in their vicinity
- The resulting largest bodies are the planetary embryos:
 - radii ~ 4000 km and masses $\sim 10^{26}$ - 10^{27} g
 - Comparable to Mercury or Mars
 - The expected time scale of the process is in the order of 10^6 yr



From planetary embryos to planets

- The final stage of planetary accretion requires the collision of embryos with residual planetesimals or with other embryos (giant impacts), until a small number of planets is formed, with masses $\sim 10^{27} - 10^{28}$ g
- This stage is called oligarchic growth
 - The growth is slower due to consumption of planetesimals in the previous, runaway stage
 - However, the largest objects still grow faster than small bodies
- Oligarchic growth continues until the large body has completely cleared the region sampled by its Hill sphere during the orbital motion
 - The time scale is of $\sim 10^7 - 10^8$ yr

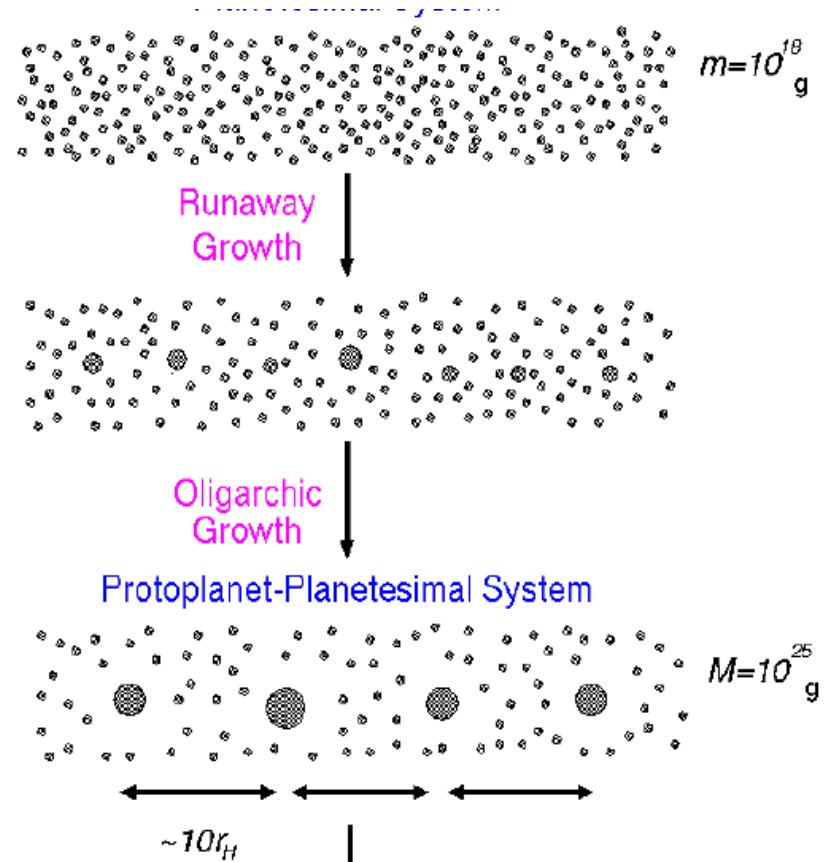
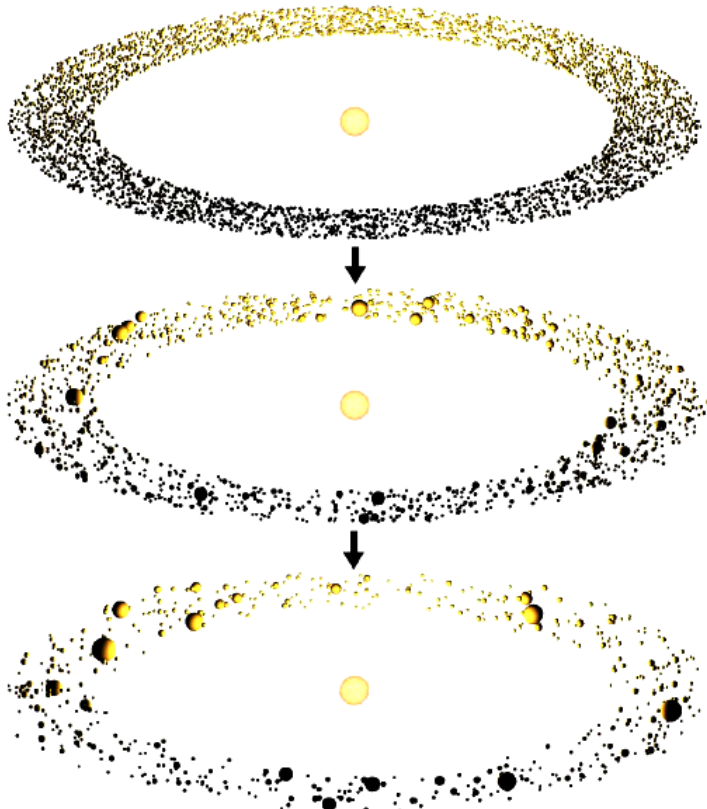


Core accretion

Planetesimals formation:

Gravitational attractions between km-sized bodies leads to **runaway grow** of the largest bodies to ~ 1000 km

Timescale $\sim 10^4$ yr



Oligarchic growth:

Larger and more massive bodies accrete to $> 10^4$ km at expenses of small bodies

Timescale \sim a few 10^5 yr

Models of terrestrial planet formation

Summary of time scales

TABLE 2.3 Stages of planetary formation according to the “standard model” (Wetherill, 1990), applied to the terrestrial planets.

Stage	Final mass (g)	Time taken (yr)	Main processes
1. Accretion of dust-sized particles into planetesimals.	10^{12} – 10^{18}	10^4	Nongravitational accumulation; particles coalesce through electrostatic forces
2. Accretion of planetesimals into planetary embryos	10^{26} – 10^{27}	10^6	Gravitational accretion aided by runaway growth
3. Accretion of planetary embryos into planets	10^{27} – 10^{28}	10^7 – 10^8	Giant impacts

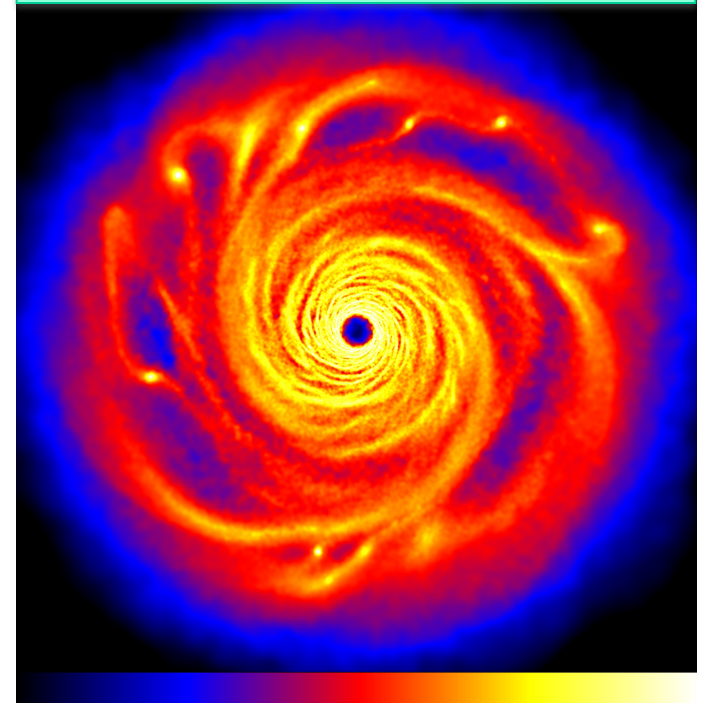
$$M_{\text{Earth}} = 5.97 \times 10^{27} \text{ g}$$

Models of giant planet formation

Boss 2004, Earth & Plan.Sci.Letters 202, 513

- Historically, two antagonistic models have been considered
 - Gravitational instability
 - Core accretion
- Gravitational instability model
 - “top-down” model of formation
 - The gaseous disk becomes unstable over short time scales, forming self-gravitating protoplanets
 - The bulk planet composition should be equal to that of the protoplanetary disk
 - Metals inside planets should sediment towards the interior
 - The outer layers should show an underabundance of metals

Simulations showing the onset of gravitational instabilities in a protoplanetary disk
Univ. California Riverside



Models of giant planet formation

- Core accretion model

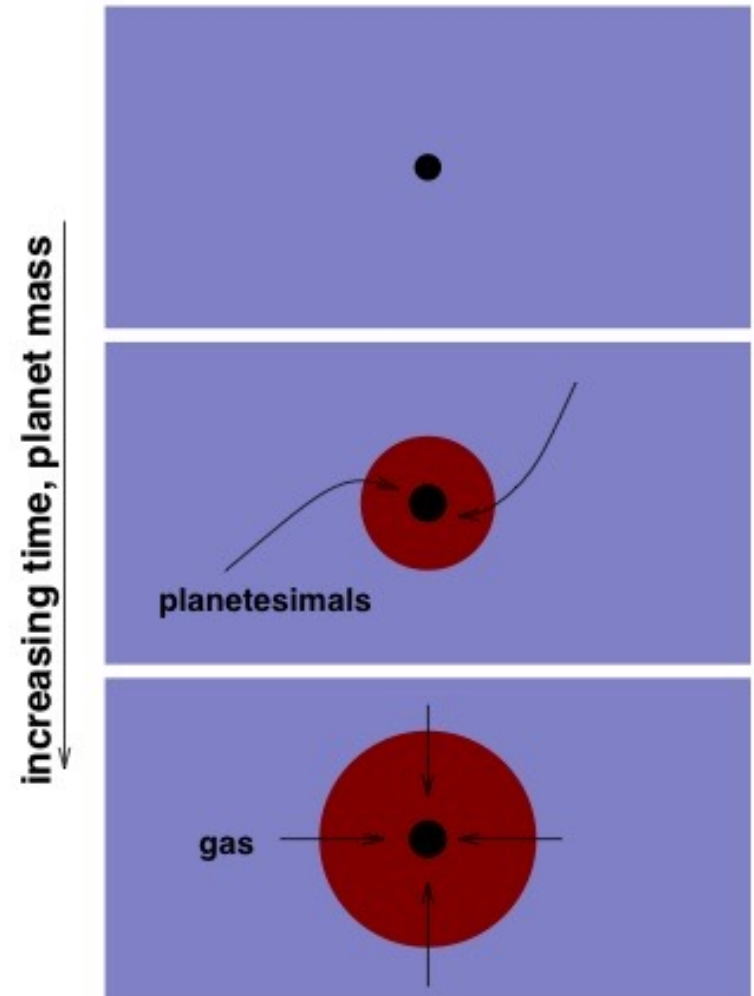
“Bottom-up” model

- A solid core with mass $M_{\text{core}} \sim 10 M_{\text{Earth}}$ is initially formed by accretion of planetesimals, as in the models of terrestrial-type planets formation

- Once formed, the solid core captures the gaseous envelope

If some of the hydrogen escapes the capture, the outer layers may show an apparent overabundance of metals

Problem: the nucleus must form very rapidly, before the gas of the protoplanetary disk is depleted, otherwise the accretion of the gaseous envelope is impossible



Models of giant planet formation

Tests to discriminate the two models

- **In the Solar System**

- The presence of a rocky core in the interiors of Saturn (and possibly Jupiter) favours the core accretion model

The existence of a rocky core in Saturn and the icy giants seems robust; in the case of Jupiter the evidence is hard to confirm

- The observed overabundances of metals in the atmospheres of the gaseous and icy giants favour the core accretion model rather than the model of gravitational instability

- **In extrasolar planetary systems**

- Part of the gaseous giants may form according to the core accretion model, as in the Solar System, but we cannot exclude that a fraction is formed by gravitational instability, given the large diversity of exoplanets

Dynamical evolution of planetary systems

- Once planets are assembled, their orbits can evolve as a result of three different types of interactions:
 - Between the planets and the gas
 - Between the planets and the planetesimals
 - With an unstable system of massive planets
- The existence of such interactions is supported by theoretical arguments and by the observed properties of the Solar System and extrasolar planetary systems
 - For instance, the last two mechanisms are supported by the great dispersion of orbital eccentricities found in exoplanets
 - We briefly discuss the interaction between the planets and the gas, which is fundamental to understand planetary migration

Interactions between the planet and the gas of the disk

- These interactions produce orbital migration due to the exchange of angular momentum between the planet and the gas
 - Planet migration was predicted well before the discovery of extrasolar planets (Goldreich and Tremaine 1980)
 - The modelization of this mechanism is quite complex
 - The migration should take place on time scales comparable to those of planetary formation and evolution of the gaseous disk
 - Ideally, one should model all these processes at the same time in order to make a self-consistent prediction
- The models predict three possible types of migration
 - Type I migration
 - Type II migration
 - Type III migration (same as Type I but planet interacts with vortex)
- Migrations are driven by resonances between the planets and the gas

Exchanges of angular momentum through resonances

- The exchange of angular momentum takes place with gas in orbital resonance with the planet
 - For circular orbits the exchange takes place in correspondence to *Lindblad* resonances
 - For the ideal case of gas moving with Keplerian velocity, *Lindblad* resonances are defined as

$$m(\Omega_g - \Omega_p) = \pm \Omega_g$$

where m is an integer; Ω_g and Ω_p are the gas and planet angular frequencies

In this ideal case the resonances are found at the radii:

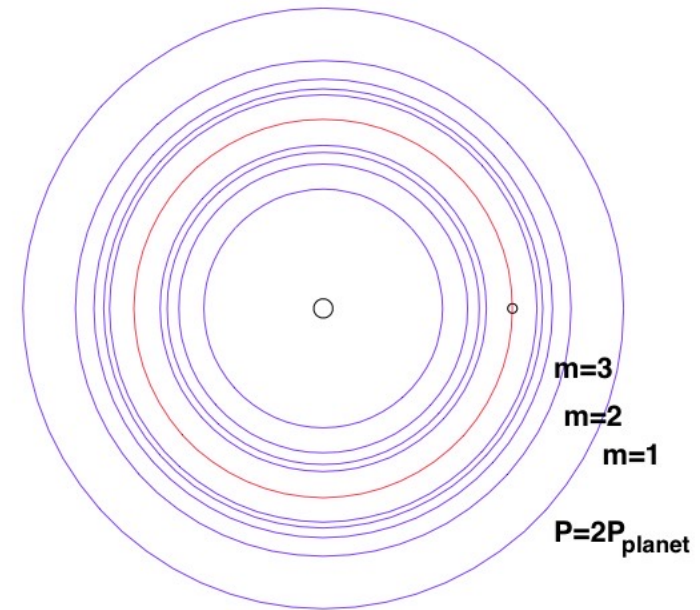
$$r_L = (1 \pm 1/m)^{2/3} r_p$$

where r_p is the orbital radius of the planet

- Depending on the value of the sign, one can have internal resonances ($r_L < r_p$) or external resonances ($r_L > r_p$)

Exchanges of angular momentum through resonances

- Internal resonances: the gas loses angular momentum while the planet gains angular momentum and migrates outwards
- External resonances: the gas gains angular momentum while the planet loses angular momentum and migrates inwards
- The net effect is counter-intuitive: the planet and the gas tend to “reject each other”, rather than being attracted by each other
- The migration takes place only if the tidal forces are stronger than the viscous forces in the disk



Some low-order Lindblad resonances are shown in the figure; high order resonances (not shown) gather in proximity of the planetary orbit (red curve)

Migration due to planet-gas interactions

- **Type I migration**

- Predicted for low-mass planets ($\sim M_{\text{Earth}}$) that cannot open a gap in the gaseous disk

Viscous forces dominate; since the gas is not removed, all the possible resonances contribute to the migration

- Calculations show that external resonances tend to dominate and the planet migrates inwards very rapidly

$\sim 10^5$ yr for a disk with mass $\sim 0.01 M_{\odot}$

- Not clear if type I migration does indeed work: it would destroy small mass planets and the cores of giant planets before the star loses the gaseous disk

Prediction not consistent with exoplanet data

However, the migration times could be longer, according to some calculations

Migration due to planet-gas interactions

- **Type II migration**

- The planet is massive enough ($\sim M_J$) to create a gap in the gaseous disk
- Tidal forces dominate over viscous forces
- Type I migration is inhibited
- Type II migration is driven by the resonances with the gas outside the gap

The process is slower due to the smaller number of resonances

The planet migrates inwards, together with the gap, on a time scale comparable to that of the disk evolution

Material keeps entering the gap

- Invoked to explain Hot Jupiters

Problem: a mechanism is required to stop the migration

