Exoplanets
Doppler method

Planets and Astrobiology (2022-2023)

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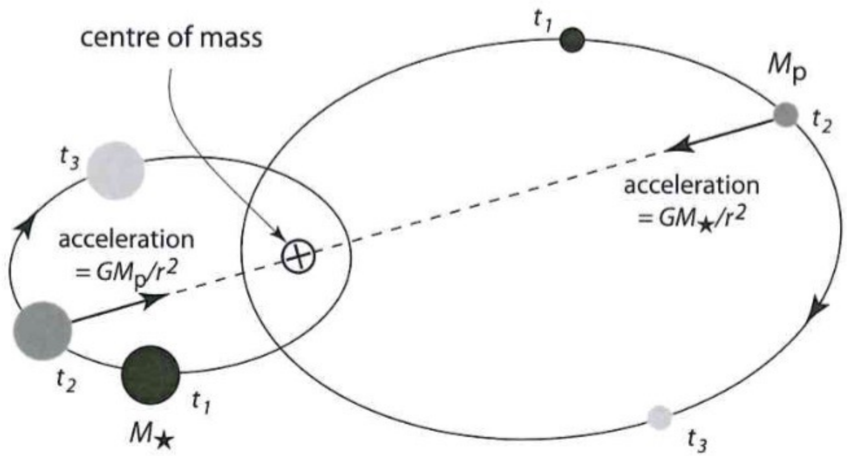
Indirect Methods:
gravitational perturbation of the stellar motion

- The star and the planet orbit around their common barycenter
  - The stellar motion, induced by the gravitational perturbation of the planet, is called "reflex motion"



Exercise:

show that the Jupiter-Sun barycenter lies outside the volume of the Sun, by 7% of the Sun's radius

Indirect Methods:
gravitational perturbation of the stellar motion

$$a_* = a\,\frac{M_p}{M_*}$$

The reflex motion of the star is proportional to M_p/M_*

This introduces an observational bias

that favours the detection of massive planets around low-mass stars

Indirect methods based on the perturbation of the stellar motion:

Radial Velocity (or Doppler)

Timing

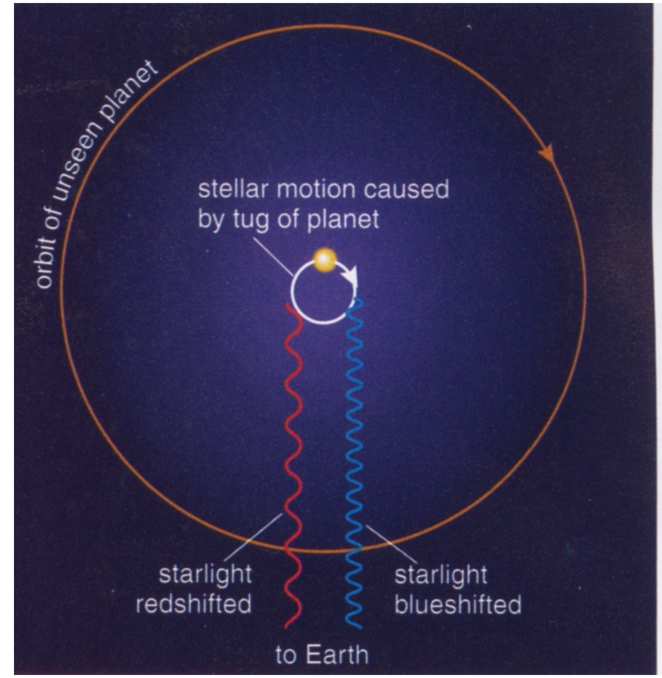
Astrometric

Doppler method (Radial velocimetry)

- Spectroscopic measurement
  - Based on the measurement of the radial velocity of the stellar reflex motion at different epochs

    The stellar radial velocity contains a variable term, $\boldsymbol{V_* \sin i}$, due to the projection of the reflex motion along the line of sight

    Thanks to the Doppler effect, the variable term $\boldsymbol{V_* \sin}$ induces a periodic shift of the photospheric lines of the stellar spectrum
- Requires high resolution spectroscopy
  - e.g., Echelle spectroscopy



$$\Delta\lambda = \lambda_{\rm obs} - \lambda_{\rm em}$$

$$v_r \simeq \left(\frac{\Delta\lambda}{\lambda_{\rm em}}\right) c$$

Masurement of the stellar radial velocity

- Based on the measurement of the centroid of stellar absorption lines

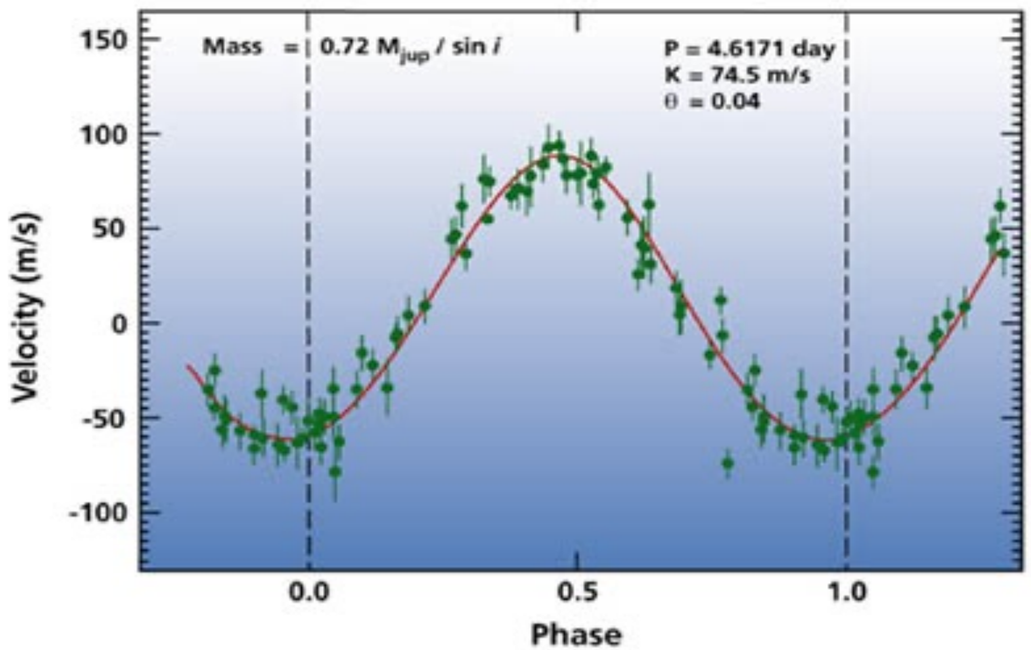
- The central position of the absorption line, λ_c, can be measured with line-fitting methods
- For unblended, unsaturated lines, a Gaussian fit generally provides an accurate determination of the centroid
- The centroid λ_c is compared with the laboratory wavelength of the electronic transition responsible for the absorption, λ_o

- In this way we calculate the radial velocity: in the non-relativistic case, the radial velocity is

$$V_r = c\,(\lambda_c - \lambda_o)/\lambda_o$$

Radial velocity curves

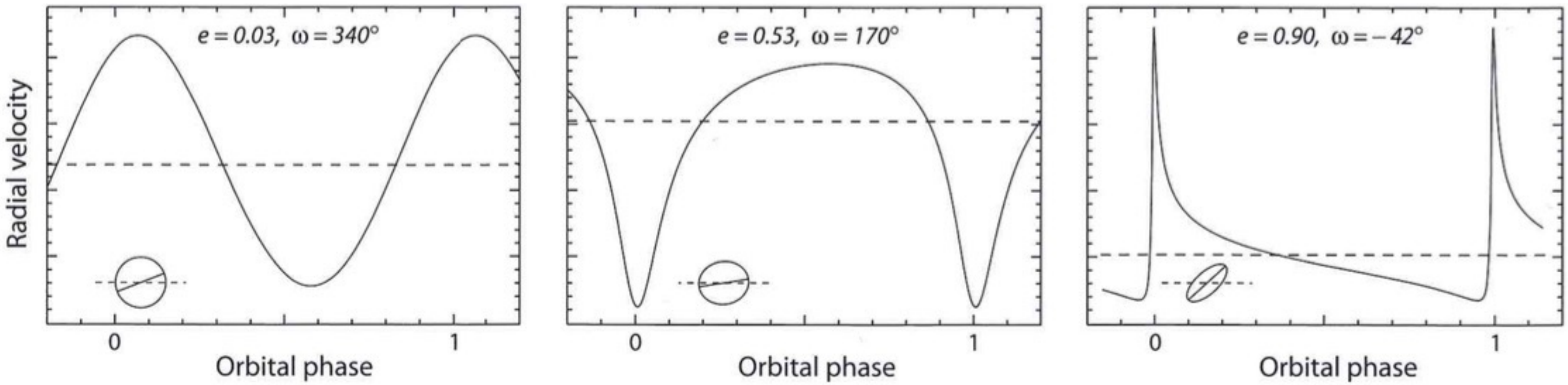
- Obtained by plotting the stellar radial velocity as a function of time
- If a periodicity is determined, data taken at different epochs are rescaled as a function of orbital phase



Radial velocity curve of 51 Peg b,
the first exoplanet detected with the Doppler method (Mayor et al. 1995)

Radial velocity curves

- For circular orbits with $M_p << M_*$ the Keplerian radial velocity curves are sinusoidal
- In general, radial velocity curves can have different shape, depending on the eccentricity, e, and the argument of the pericenter, ω



Examples of radial velocity curves

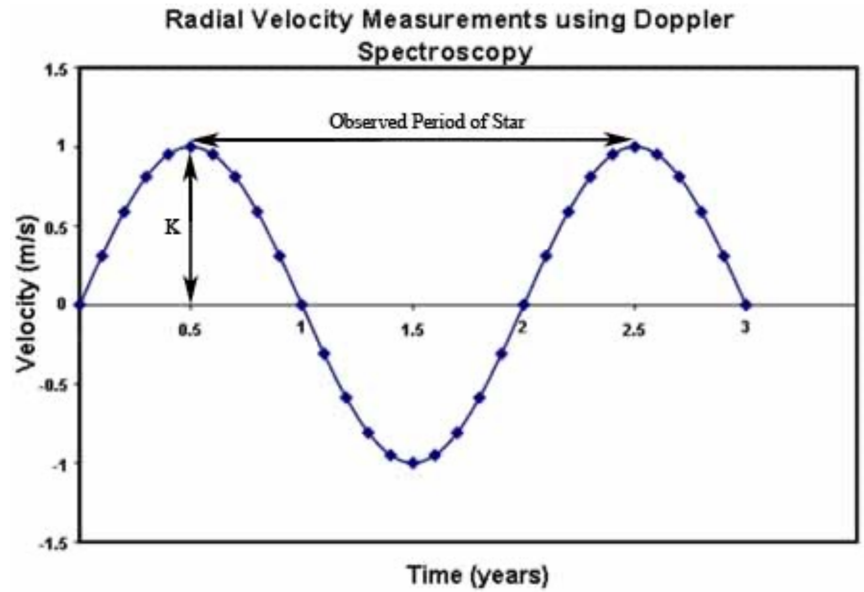
left: HD73256 - center: HD142022 - right: HD4114

dashed lines: radial velocity of the barycenter

- Parameters that can be derived from the radial velocity curve

– Semi-amplitude, K, and period, P

The amplitude corresponds to the variation of $\boldsymbol{V_* \sin i}$ during one orbital period



– The semi-amplitude is given by

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin i}{(M_\star + M_p)^{2/3}} \frac{1}{(1-e^2)^{1/2}}$$

The above expression for *K* can be used to estimate the expected effect for the Solar System planets

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Table I
Radial Velocity Signals of the Planets

Planet	M_p (M_J)	R (AU)	P (years)	$\Theta_\star$ at 10 pc (mas)	*K* ($m s^{-1}$)
Mercury	1.74E-4	0.387	0.241	6.4E-6	0.008
Venus	2.56E-3	0.723	0.615	1.8E-4	0.086
Earth	3.15E-3	1.000	1.000	3.0E-4	0.089
Mars	3.38E-4	1.524	1.881	4.9E-5	0.008
Jupiter	1.0	5.203	11.86	0.497	12.4
Saturn	0.299	9.54	29.46	0.273	2.75
Uranus	0.046	19.18	84.01	0.084	0.297
Neptune	0.054	30.06	164.8	0.156	0.281
Pluto	6.3E-6	39.44	247.7	2.4E-5	3E-5

Selection effects

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin i}{(M_\star + M_p)^{2/3}} \frac{1}{(1-e^2)^{1/2}}$$

For a given stellar mass, the amplitude of the reflex motion scales as $M_p P^{-1/3}$

- Easier to detect massive planets with short orbital period (i.e., small semimajor axis)

For a given planetary mass, the amplitude of the reflex motion scales as $M_*^{-2/3}$

- Easier to detect planets around low-mass stars (e.g. M type stars)
- Low-mass stars are intrinsically fainter, but more numerous, than stars of higher mass

The amplitude of the reflex motion is larger when the line of sight is aligned with the orbital plane ($\sin i \cong 1$)

- If orbits are oriented at random, this selection effect does not introduce a bias on the physical properties of the planetary systems

- Derivation of orbital and planetary parameters with the Doppler method
  - The radial velocity curves provides the period, P, from which the semimajor axis a is inferred using the third Kepler's law
  - The mass of the star is derived from a spectroscopic study combined with a model of stellar structure
  - By fitting the radial velocity curve one can obtain the eccentricity, e, and the argument of the pericenter, ω
  - In this way, from the semiamplitude K one can infer a lower limit on the planet mass

$$M_p \sin i = K \left(\frac{P}{2\pi G}\right)^{1/3} (M_\star + M_p)^{2/3} (1 - e^2)^{1/2}$$

- The orbital inclination

  – Systems with high inclination give a higher radial radial velocity signal and, in this sense, are more likely to be detected

  – Statistically, the correction term **sin *i*** is not very large

  – The statistical probability that the orbital inclination is within an arbitrary range (i_1, i_2) is (Fischer et al. 2014)

$$\mathcal{P} = |\cos(i_2) - \cos(i_1)|$$

  – There is roughly an 87% probability that random orbital inclinations are between 30 and 90, or equivalently, an 87% probability that the true mass is within a factor of 2 of the minimum mass $\boldsymbol{M_\mathrm{p}}$ **sin *i***

  – In any case, a safe determination of the mass requires an independent measurement of the orbital inclination, *i*

Cross correlation spectroscopy

Information on the instantaneous Doppler shift is contained in the many thousands of absorption lines present in the high-resolution optical spectra of solar-type stars

This information can be concentrated into a few parameters
by cross-correlating the observed spectrum, *S*,
with a template of the expected stellar spectrum, *M*,
in the radial velocity space, *v*

In practice, one has to determine the value of ϵ that minimizes

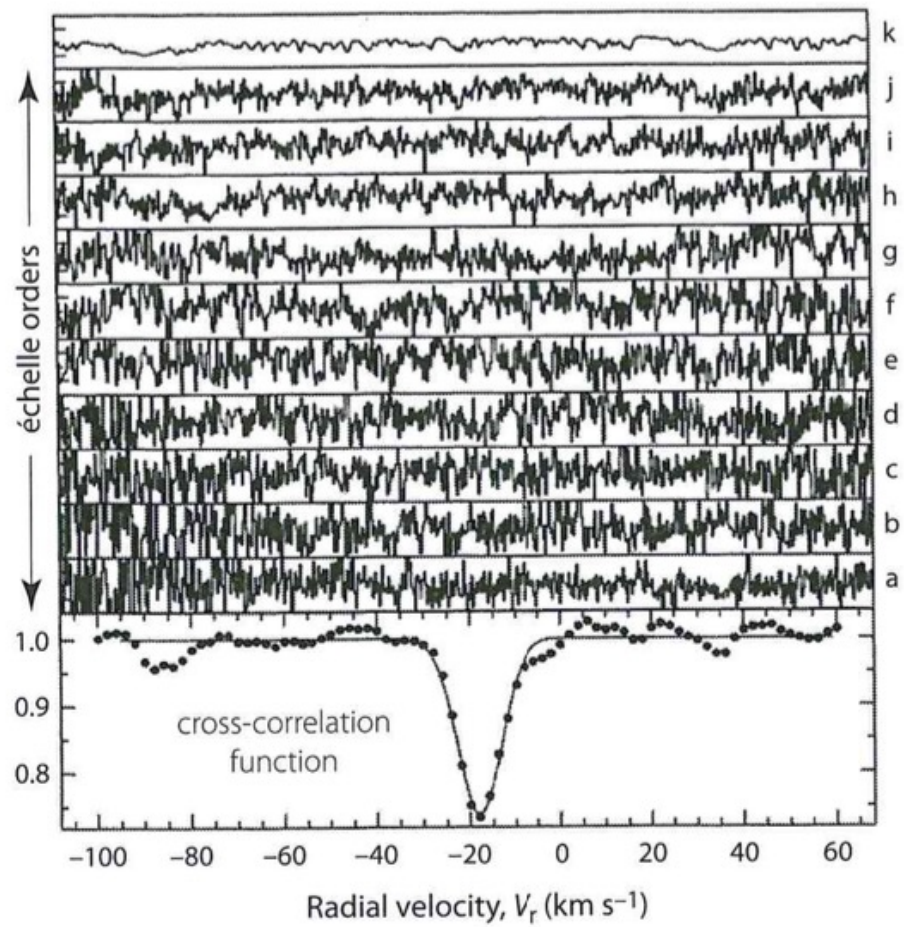
$$C(\epsilon) = \int_{-\infty}^{+\infty} S(v)\, M(v-\epsilon)\, dv$$

Thanks to the use of thousands of absorption lines, the cross correlation is efficient even at low signal-to-noise ratio

Cross correlation spectroscopy

Example cross-correlation function for a K0 III star with S/N ~ 1. Observations at R=40000 in 10 echelle orders, each covering ~ 4 nm. About 1000 lines match the template. The resulting cross-correlation function is shown at the bottom.

(Queloz 1995)



Doppler method

- Technological requirements

  Extremely stable spectrograph
  – Long term stability in temperature (possibly years)
  – The spectrograph is closed in a separate room/container, fed by optical fibres

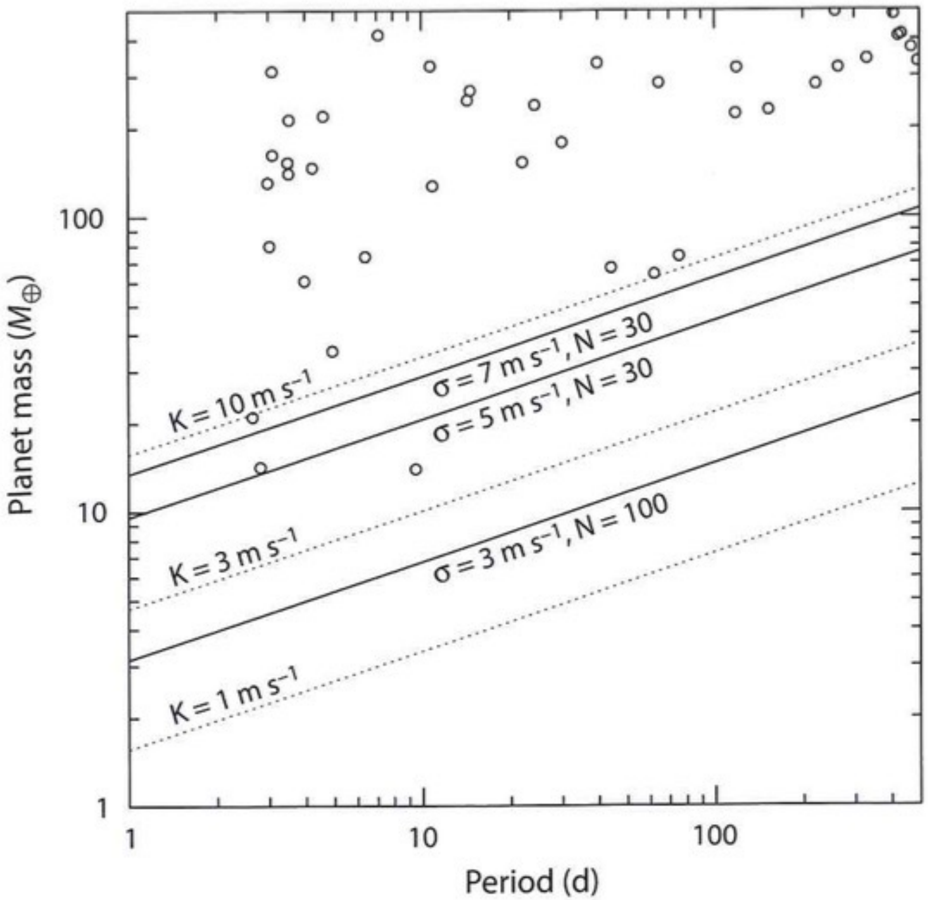
  High accuracy of wavelength calibration
  – Control of the optical paths of the stellar and wavelength calibration spectra
  – Thorium-argon calibration
    Classic observations
  – Laser frequency combs
    Cover the entire wavelength range of interest with a series of calibration lines of uniform spacing and intensity and accurately known wavelengths determined by fundamental physics

- Mass limit of detectability

  Minimum mass limit for a 50% detection threshold as a function of the number of observations, N, and the combined error, σ, for M_*=1 M_{sun}

  In addition to the measurement error, the "stellar jitter" (activity in the stellar atmosphere and stellar oscillations) contributes to the combined error σ

  With increasing measurement accuracy, stellar jitter becomes the most severe limit of application of the method



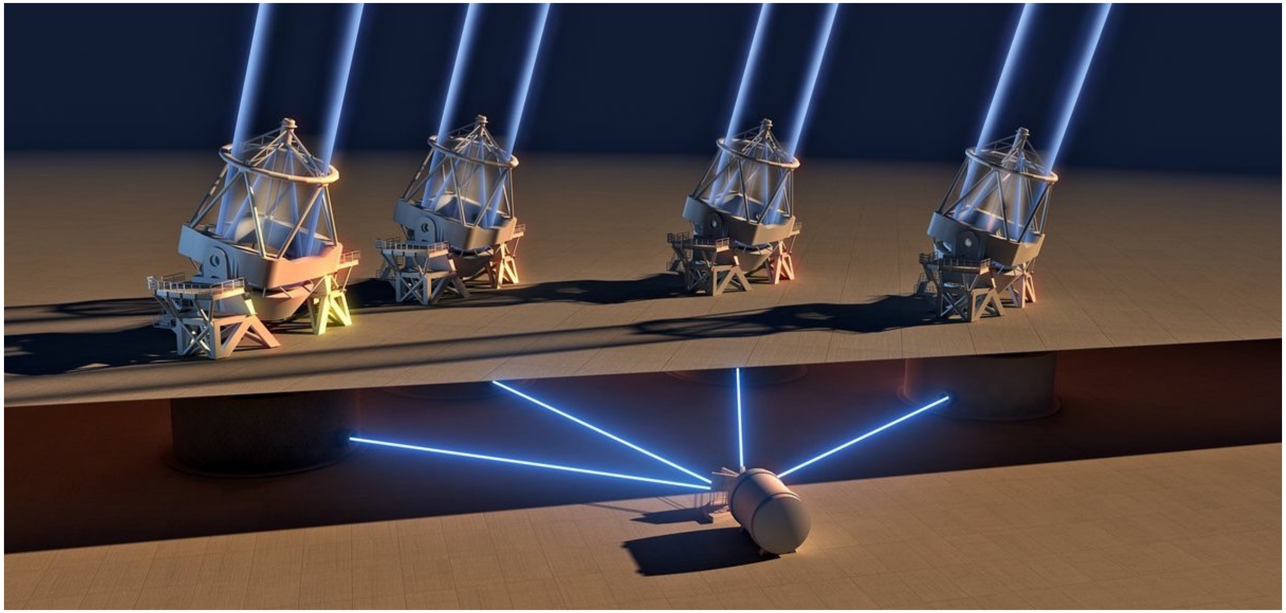
Doppler method: examples of spectrographs

- HARPS
  - High Accuracy Radial velocity Planet Searcher
  - Telescope: ESO 3.6m La Silla (Cile)
    - http://www.eso.org/sci/facilities/lasilla/instruments/harps/
  - Has played a pioneering role, and is still effective, in the search for extrasolar planets
  - Estremely stable, with current limits of radial velocity accuracy ~40 cm/s for planets orbiting relatively bright stars (V~12)
- HARPS-north
  - Based on the HARPS spectrograph, installed in 2012 at the Italian national telescope TNG (3.5m, La Palma, Spain)
  - Extends the capabilities of HARPS to the northern hemisphere

- ESPRESSO
  - Echelle SPectrograph for Rocky Exoplanet and Stable Spectroscopic Observations
    - http://www.eso.org/sci/facilities/develop/instruments/espresso.html
  - Extremely stable, recently installed at the combined focus of the ESO VLT (8m x 4)

    Radial velocity accuracy: < 10 cm/s

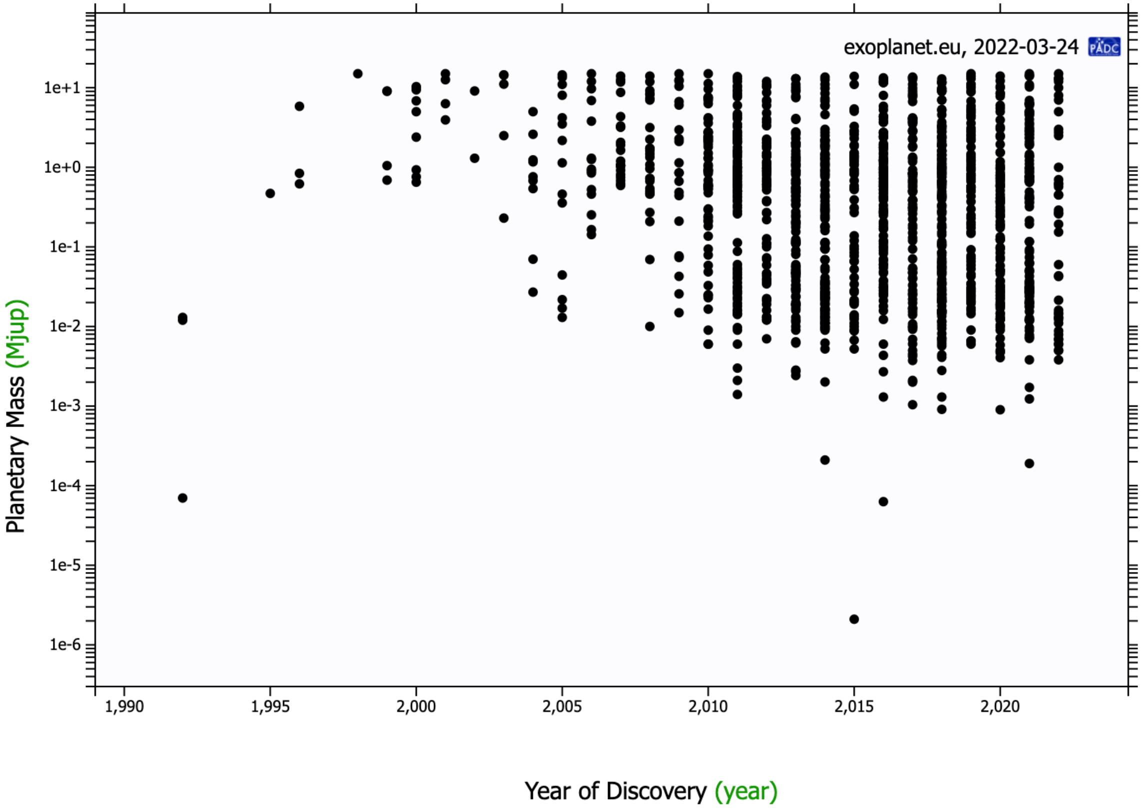
    Fainter stars than observable with HARPS
  - Detection of terrestrial-type planets at ~1 AU around solar-type stars



Doppler method

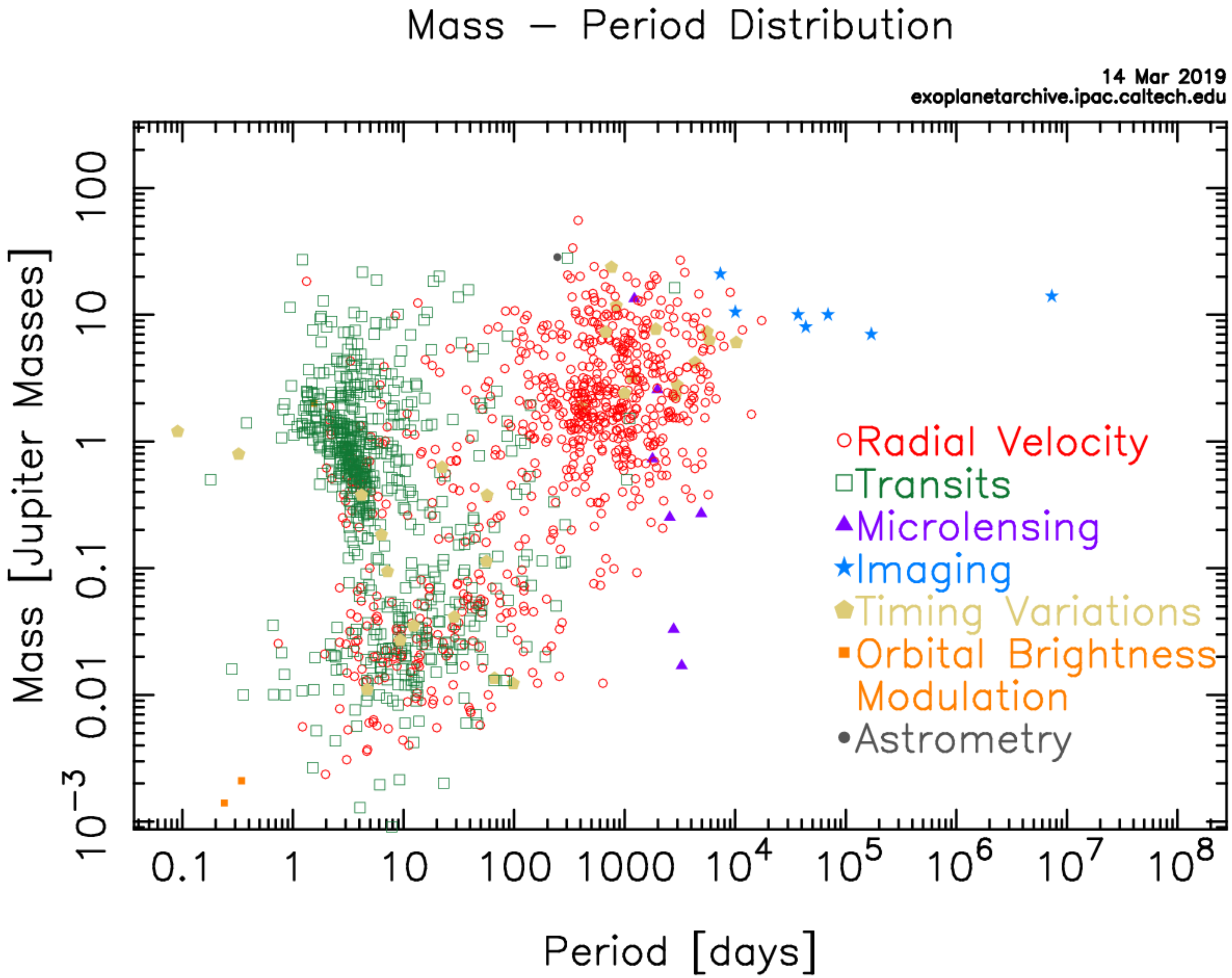
- Very effective
  - Between 1995 and 2012 most exoplanets have been discovered with the Doppler method
  - Currently (2022):

    991 planets / 737 planetary systems / 174 multiple planetary systems



Doppler method

comparison with other methods



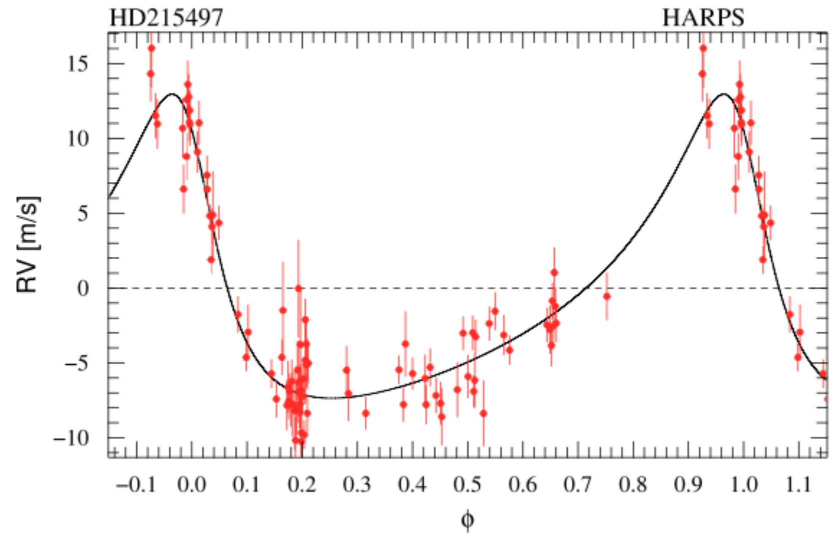
Detection of multiple planetary systems with the Doppler method

- The total radial velocity signal due an orbiting system of n_p planets is assumed to result from the independent reflex motions due to each planet separately

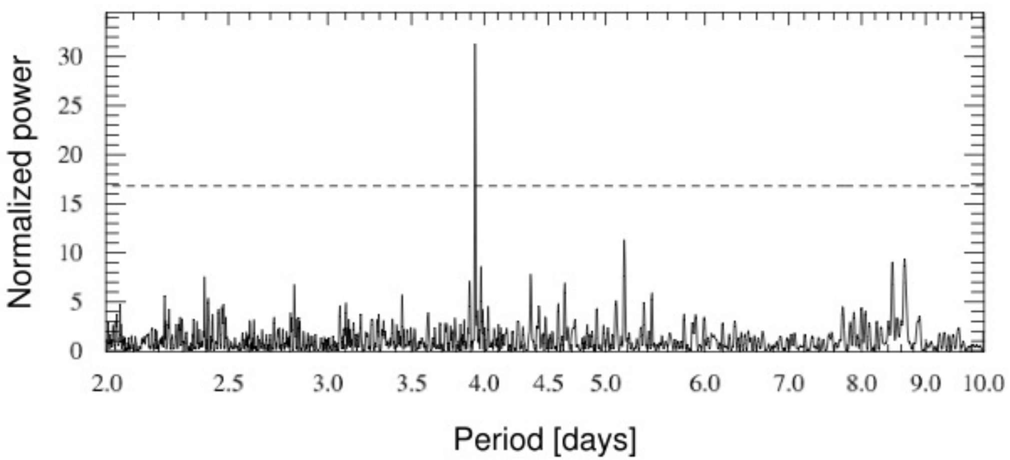
- Procedure:
  - Make a Keplerian fit of the most intense signal of the radial velocity curve
  - Subract the Keplerian fit to the experimental data and search for more periodic signals in the residuals
  - If the periodogram (Fourier transform) of the residuals shows a peak above the noise level, a further Keplerian fit is performed
- The procedure can be iterated if the periodogram of the residuals shows additional significant peaks

Example:
Two-planet system orbiting HD215497 (Lo Curto et al. 2010)

Detection of the planet that yields the main signal (P= 568 d)

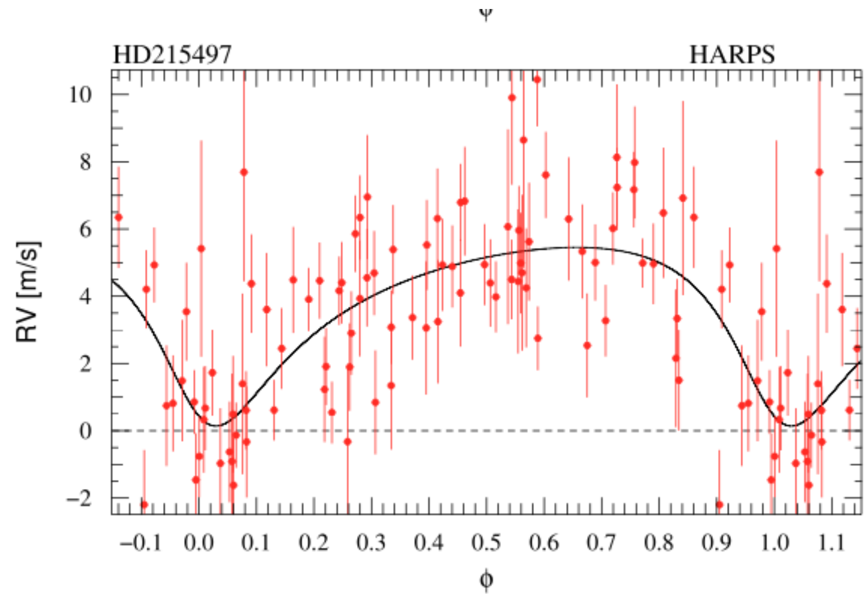


The periodogram of the residuals shows a peak at P=3.93 d, well above the noise level



Example:
Two-planet system orbiting HD215497 (Lo Curto et al. 2010)

Keplerian fit of the residuals, using the orbital parameters determined from the previous step



Final results

Parameter	HD 215497b	HD 215497c
P [days]	3.93404 ± 0.00066	567.94 ± 2.70
T [JD-2 400 000]	54 858.95 ± 0.37	55 003.48 ± 5.15
e	0.16 ± 0.09	0.49 ± 0.04
ω [deg]	96 ± 34	45 ± 4
K [m s^{-1}]	2.98 ± 0.34	10.10 ± 0.65
V [km s^{-1}]	49.3107 ± 0.0006	
$m \sin i$ [$M_{\rm Jup}$]	0.02	0.33
$m \sin i$ [$M_{\rm Earth}$]	6.6	104.3
a [AU]	0.047	1.282

Doppler method
future projects

- CODEX
  - Instrument concept for the European ELT (Extremely Large Telescope)
  - It aims to detect the expansion of the Universe directly, by measuring the Doppler shift of high-redshift quasar Ly-α absorption lines as a function of time
  - The experiment targets a Doppler accuracy of ~0.02 m/s mantained over several <u>decades</u>
  - The design has developed from HARPS and will incorporate all the state-of-the-art technology (e.g. laser comb, etc.)
  - CODEX will have direct application to exoplanet studies
  - Stellar jitter would definitely become the major limitation