# Observational constraints for models of planetary formation

Planets and Astrobiology (2019-2020)

G. Vladilo and S. Ivanovski

# Observational constraints of Solar System formation

# Dynamical constraints

- Orbital motions of the planets
  - Most orbits are coplanar with the Sun equator
  - Most orbits are prograde with respect to solar rotation

- These characteristics suggest a common origin of the Sun and the planets through a process that has preserved the angular momentum
  - Consistent with an origin in a "solar nebula" with a disk shape

Hypothesis already advanced, in schematic form, by Laplace

As a result of the contraction of the nebula, a disk is formed which provides the material for the planetary formation

# The angular momentum of the Solar System

$$\boldsymbol{L} = \boldsymbol{r} \times m\boldsymbol{v}$$

- Rotational angular momentum

$$L^{(rot)}_{Sole} = 1{,}6 \bullet 10^{40}\ (kg \bullet m^2)/s$$
$$L^{(rot)}_{Giove} = 4{,}5 \bullet 10^{38}\ (kg \bullet m^2)/s$$
$$L^{(rot)}_{Saturno} = 7{,}7 \bullet 10^{37}\ (kg \bullet m^2)/s$$

- Orbital angular momentum of the planets

$$L^{(orb)}_{Giove} = 1{,}9 \bullet 10^{43}\ (kg \bullet m^2)/s$$
$$L^{(orb)}_{Saturno} = 7{,}9 \bullet 10^{42}\ (kg \bullet m^2)/s$$

- Most angular momentum of the Solar System is concentrated in the orbital angular momentum of giant planets

# The problem of the angular momentum

- In the nebular hypothesis, one would expect that most of the angular momentum was accumulated in the Sun

- The fact that 98% of the angular momentum lies in the orbital angular momentum of giant planets poses, in principle, a problem for the simple scenario of a common origin from the Solar Nebula

- We must invoke that the Sun has lost angular momentum

  Possibly:

  In the protostellar phase, when part of the accreted momentum was used to energize bipolar outflows/jets

  During the main-sequence evolution: part of the rotational energy is transformed in magnetic fields and solar wind through different types of mechanisms (e.g. the dynamo mechanism)

# Constraints on the solar nebula: Density gradient and minimum mass

- Radial distribution of the mass in the solar nebula
  - Approximate estimate based on the following hypothesis:

    Each planet, at the epoch of its formation, has accreted all the mass available in its region of influence in the nebula

    Rocky planets have lost their volatile components: the mass of the nebula in their region of influence should be scaled from the mass measured in the refractory component

**THE DISTRIBUTION OF MASS IN THE PLANETARY SYSTEM AND SOLAR NEBULA**

S. J. WEIDENSCHILLING

*Dept. of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D.C., U.S.A.*

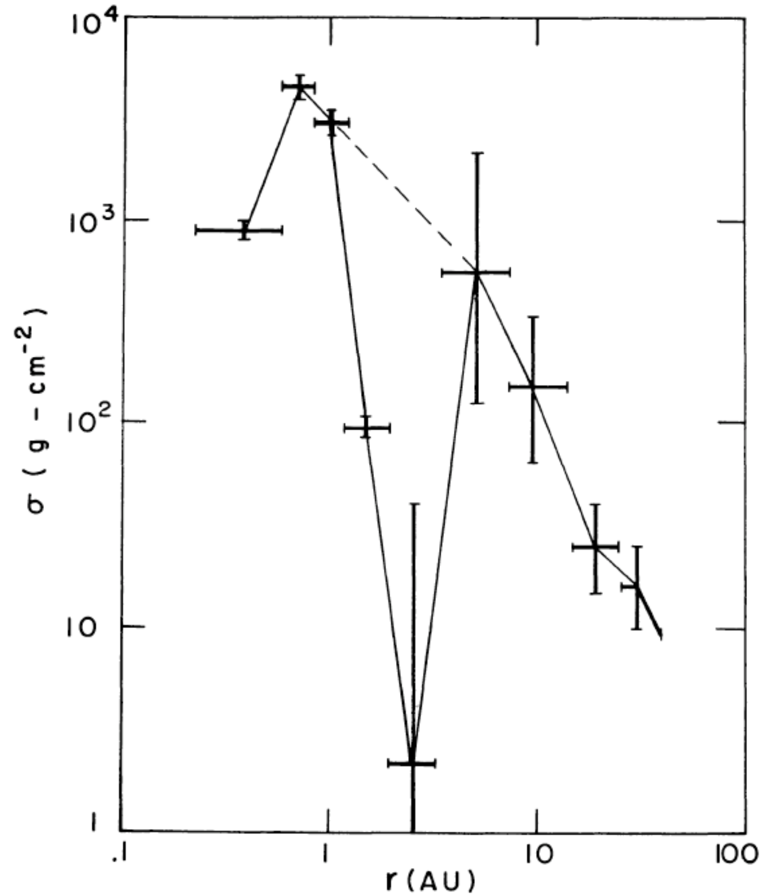
(Received 19 February, 1977)

# Constraints on the solar nebula: Density gradient and minimum mass

- Radial distribution of the nebular mass
  - The density profile obtained with the above hypothesis provides a minimum surface density of the solar nebula

    The profile has a negative density gradient, in agreement with the expectation that the nebula was thicker in the central regions

    By integrating the profile one obtains the "minimum mass solar nebula (MMSN): 0.01 e 0.07 $M_\odot$
- The profile shows anomalies, possibly related to the loss of mass in the asteroid belt
- The MMSN is important for historical reasons
  - Nowaday we know that it is oversimplified since it does not take into account the possibility of planet migration



# Radial gradients of refractory and volatile elements

Volatile and refractory elements are distinguished according to their <u>condensation temperature</u> $T_{cond}$

- Temperature at which an element condenses out of a cooling gas of given composition and pressure
- Each element can condense in different chemical compounds

Volatiles: $T_{cond} < 10^3$ K

Refractories: $T_{cond} \gtrsim 10^3$ K

| Substance | Formula | Temperature of condensation[a] (K) |
|---|---|---|
| *ionic substances* | | |
| corundum | $Al_2O_3$ | 1758 |
| perovskite | $CaTiO_3$ | 1647 |
| spinel | $MgAl_2O_4$ | 1513 |
| nickel–iron metal | Ni, Fe | 1471 |
| pyroxene (diopside) | $CaMgSi_2O_6$ | 1450 |
| olivine (forsterite) | $Mg_2SiO_4$ | 1444 |
| alkali feldspars | $(Na,K)AlSi_3O_8$ | <1000 |
| troilite | FeS | 700 |
| hydrated minerals[b] | (variable) | 550–330 |
| *molecular substances* | | |
| water | $H_2O$ (as an ice) | 180 |
| ammonia | $NH_3.H_2O$ (ice) | 120 |
| methane | $CH_4.6H_2O$ (ice) | 70 |
| nitrogen | $N_2.6H_2O$ (ice) | 70 |

[a] The temperatures of condensation given are those that would occur if the pressure in the nebula had been about $10^{-3}$ bar ($10^2$ Pa). At lower pressures, the condensation temperatures would have been reduced slightly.

[b] Hydrated minerals are chiefly silicates with OH or $H_2O$ in their formulae.

## Radial gradients of refractory/volatile components in the Solar System

- Inner solar system: the bulk ingredients of rocky planets are refractories
  - Silicate mantles and iron cores: $T_{cond} > 10^3$ K
- Outer solar system: the bulk ingredients of giant planets are volatiles
  - Gasesous and icy volatiles and ices: $T_{cond} \sim 10^2$ K

- Similar differences between the inner and outer solar system are found in minor bodies:
  - Asteroids, closer to the Sun, are in large part rocky
  - Comets, at larger distances, are mostly icy

# Radial gradients of refractory and volatiles elements:

## implications for the formations of the Solar System

- The radial gradient of the refractories and volatiles suggests the existence of a temperature gradient in the solar nebula, consistent with what expected from the radiative heating of the nebula by the protosun
- The distance at which (icy) volatiles start to predominate with respect to refractories is called the "snow line" (or "ice line")
  - Simplest definition of "snow line":

    Distance from the Sun at which a body heated by solar radiation and cooled by black body emission attains an equilibrium temperature that allows ices to be formed ($H_2O$, $NH_3$, $CH_4$)
  - The real calculation of the ice line is more complicated since one must also take into account the radiative transfer through the protoplanetary disk and other physical processes at the level of microscopic solids

# The ice line

Example: calculation of the ice line given the solar luminosity, $L_\odot$, the condensation temperature of ice, $T_{ice}$, and the ice albedo, $A$

General equations:

$$\sigma T^4_{eff} = \tfrac{1}{4} S (1-A)$$

$$S = L / (4 \pi d_{ice}^2)$$

Solutions for the solar system:

$$d_{\rm ice} \simeq \frac{1}{4 T^2_{\rm ice}} \sqrt{\frac{L_\odot (1-A)}{\pi \sigma}}$$

$$d_{\rm ice}[{\rm AU}] \simeq \left( \frac{272}{T_{\rm ice}\,[{\rm K}]} \right)^2 \sqrt{(1-A)}$$

Gives $d_{ice} \sim 2.3$ AU for $T_{ice}$=150 K and $A$=0.5

## Processes leading to the radial gradient of volatiles/refractories

- Condensation of solids *in situ* in the nebula

  the refractory component can condense at all distances from the protosun, while the volatile components can condense only beyond the ice line

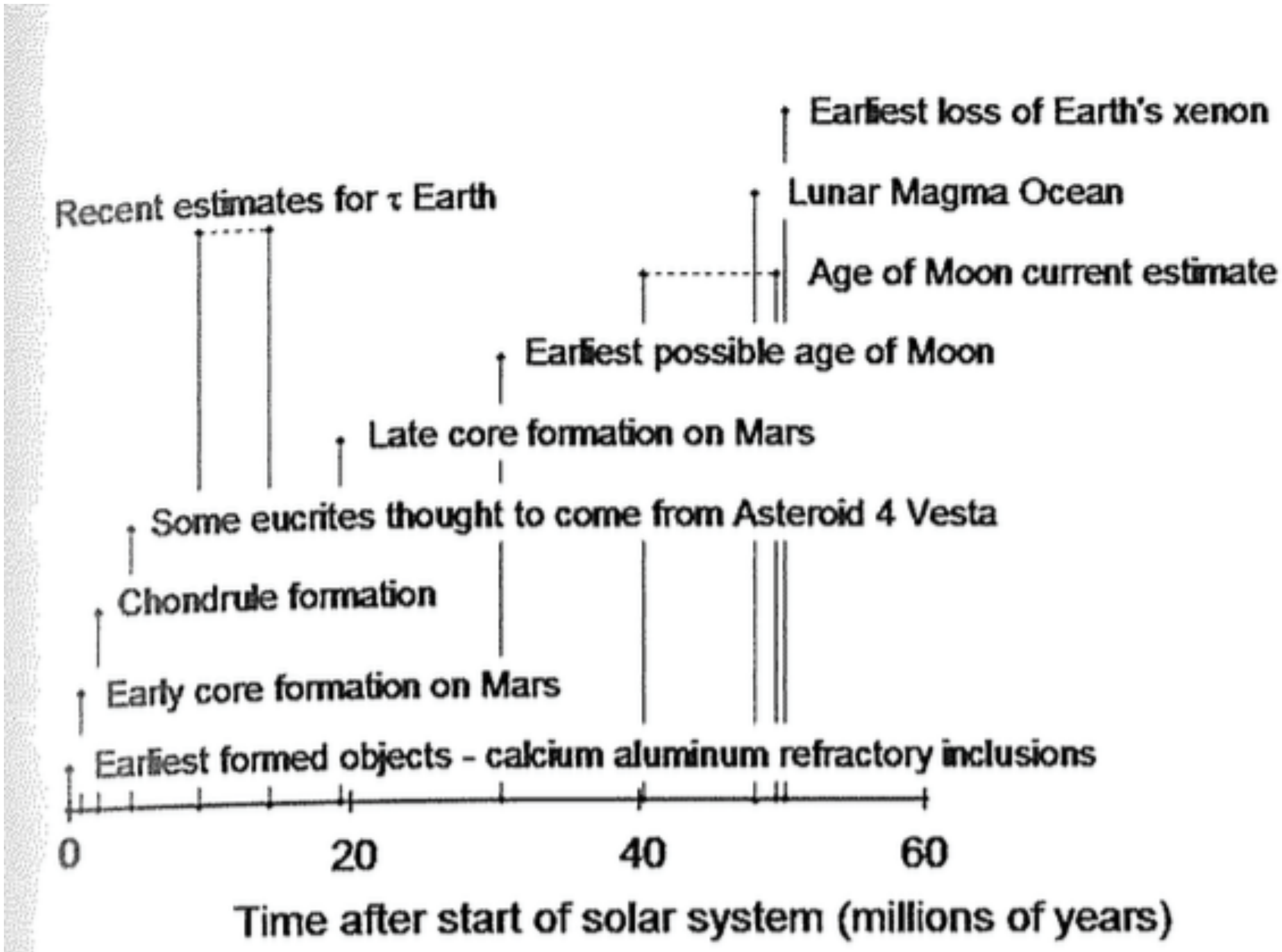
- Acquisition of solids (dust grains) previously formed the interstellar medium

  the refractory components acquired from the interstellar medium can approach the vicinity of the sun, while the volatile component would sublimate

  the icy component acquired from the interstellar medium can persist only far from the protosun

# Time scales of formation of the inner Solar System

## Based on radiodating techniques




Encyclopedia of the Solar System

# Observational constraints on planetary formation obtained from studies of exoplanet systems

- Hot Jupiters

  The presence of gaseous giants in the proximity of the star requires mechanisms of radial migration of giant planets

- Peak in the distribution of orbital periods at $P \sim 3$ days

  Requires mechanisms capable of stopping the migration towards the star

  Cases may exist in which the migrating planet is not stopped and merges with the central star

- Dispersion of the values of orbital eccentricities and inclinations

  Requires mechanisms of gravitational scattering of planetary orbits

# Observational constraints on planetary formation obtained from studies of exoplanet systems

- Diversity of possible types of planets

  Requires models of planetary formation capable of producing, "super-Earths", "ocean planets, "inflated Jupiters"

- Occurrence rate versus metallicity of the host star

  If interpreted as a primordial effect, rather than self-enrichment, requires exploring the dependence on metallicity of the processes involved in planetary formation

  - The dust-to-gas ratio is expected to scale with the level of metallicity
  - The accretion of solids in the protoplanetary disk scales with the local dust-to-gas ratio, which may be enhanced with respect to the interstellar value ~0.01

# Appendix: Radioactive dating

## Isochron methodology

We call $P(t)$ and $D(t)$ the time-dependent abundance of the parent and daughter isotopes. The time-dependence of the parent isotope is

$$P(t) = P(0)\, e^{-kt} \tag{1}$$

where $k$ is the decay constant (the half-time of decay is $\tau = \ln 2/k$).

The sum of the parent and daughter isotopes is constant with time:

$$D(t) + P(t) = D(0) + P(0) \tag{2}$$

If there is a stable isotope D' of the daugther element D, and we presume that D' is constant throughout the process, then

$$\frac{D(t) + P(t)}{D'} = \frac{D(0) + P(0)}{D'} \tag{3}$$

# Appendix: Radioactive dating

## Isochron methodology

By replacing (1) in the above expression we obtain

$$\frac{D(t)}{D'} = \frac{P(t)}{D'}\left[e^{kt} - 1\right] + \frac{D(0)}{D'} \tag{4}$$

If we call $y = D(t)/D'$ and $x = P(t)$, we have

$$y = x\left[e^{kt} - 1\right] + y(0) \tag{5}$$

By plotting $y$ versus $x$ the slope will be $dy/dx = e^{kt} - 1$ and the age

$$t = \frac{1}{k}\ln\left[1 + \frac{dy}{dx}\right] \tag{6}$$

In this way, if there are samples of the same age, we will obtain an isochrone from which the age can be derived.

# Appendix: Radioactive dating

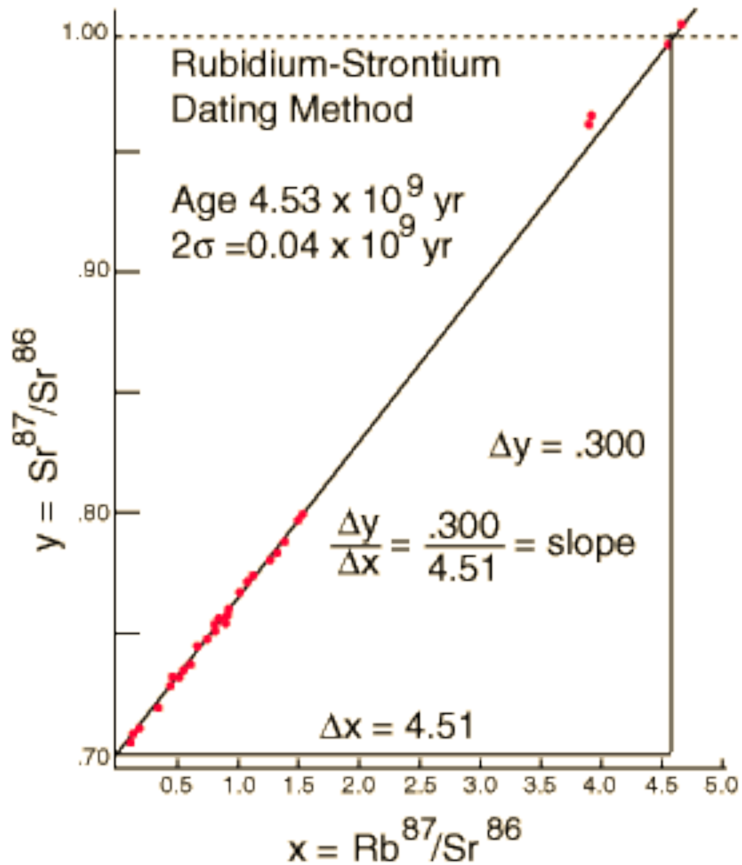
## Isochron methodology

**Example**

P = $^{87}Rb$

D = $^{87}Sr$

D' = $^{86}Sr$




$^{86}Sr$ is of non-radiogenic origin and can be used as a reference concentration to imply the original concentration of $^{87}Sr$.