XXXII Canary Islands Winter School of Astrophysics

Galaxy clusters in the local Universe

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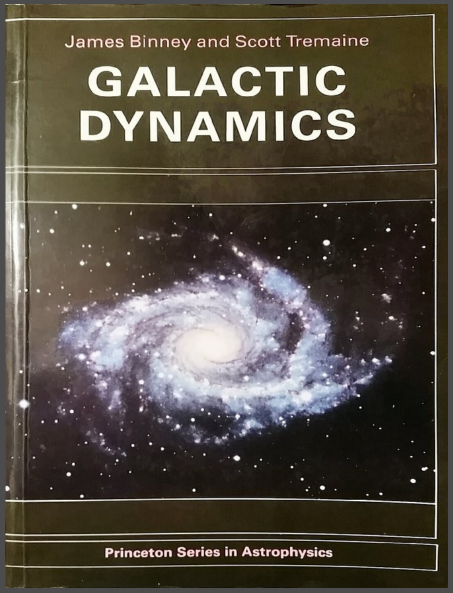
Lecture 5 (part 1):

Masses & mass profiles

Based on:

Binney & Tremaine (1987), Chapters 4.1, 4.2, 4.3



Pratt et al. (2019), Sections 2.3, 2.5, 3

Published: 28 February 2019

The Galaxy Cluster Mass Scale and Its Impact on Cosmological Constraints from the Cluster Population

G. W. Pratt, M. Arnaud, A. Biviano, D. Eckert, S. Ettori, D. Nagai, N. Okabe & T. H. Reiprich

Space Science Reviews **215**, Article number: 25 (2019) | Cite this article

Kneib (2008):

J.-P. Kneib: *Gravitational Lensing by Clusters of Galaxies*, Lect. Notes Phys. **740**, 213–253 (2008)
DOI 10.1007/978-1-4020-6941-3_7 © Springer Science+Business Media B.V. 2008

Additional readings:

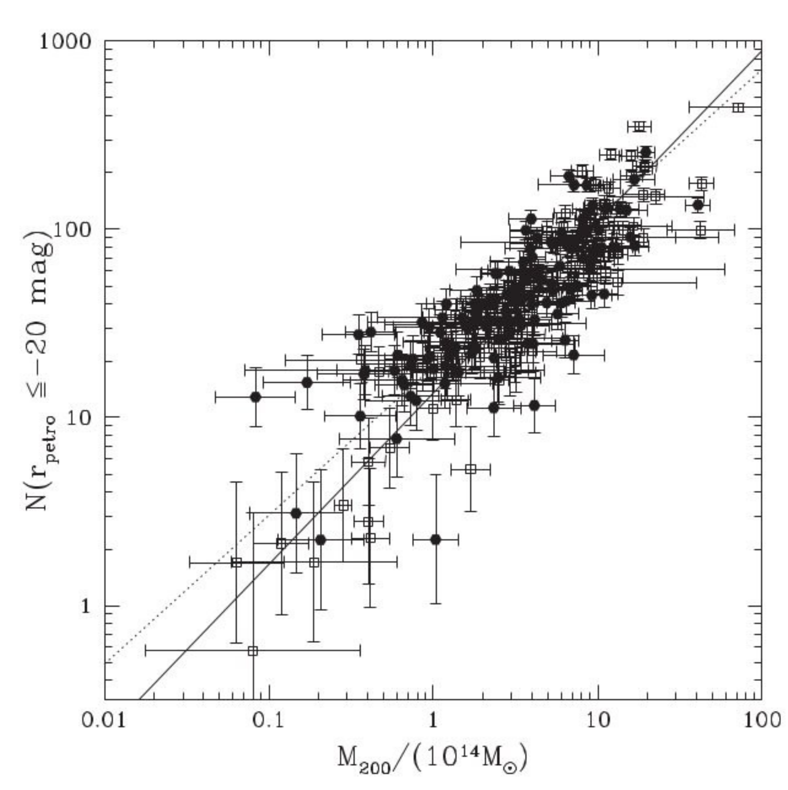
Girardi et al. (1998), ApJ, 505, 74 *(on the virial theorem)*
Mamon, AB, Boué (2013), MNRAS, 429, 3079 *(the MAMPOSSt method)*
Diaferio (1999), MNRAS, 309, 610 *(Caustic method)*

Masses & mass profiles

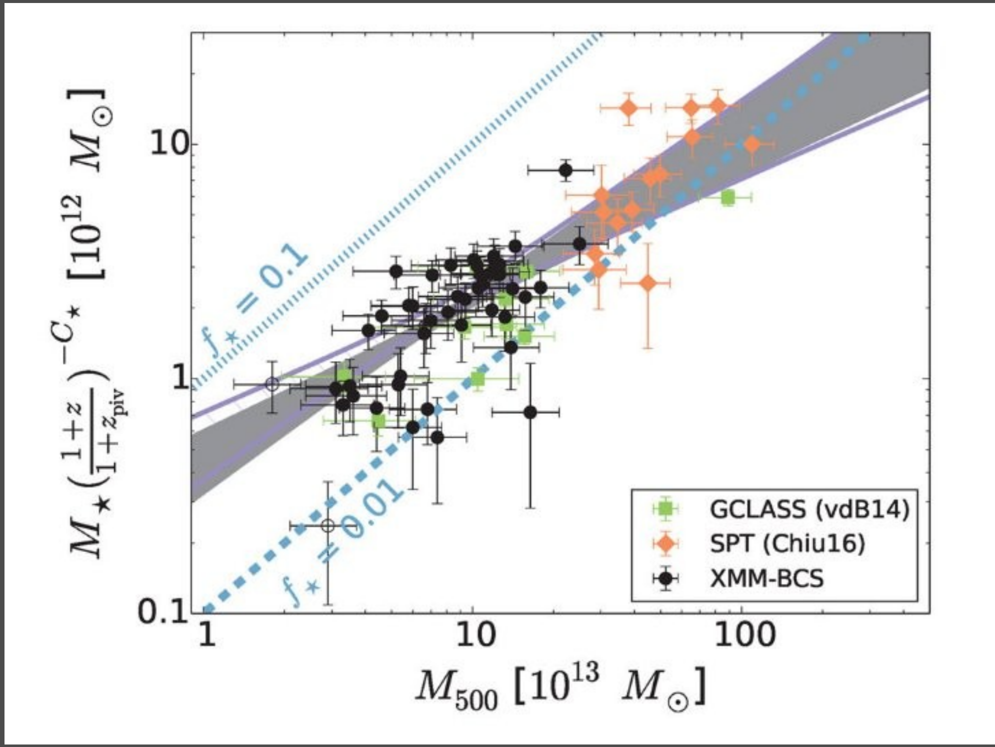
Galaxies

If only photometric information is available, define simple mass proxies:

- number of galaxies above a given stellar mass or absolute magnitude ("**richness**")
- **total luminosity** or **stellar mass**



Popesso, AB et al. (2007):
richness vs mass (from kinematics)



Chiu et al. (2016):
total stellar mass vs mass (from X-ray hydrostatic)

Masses & mass profiles

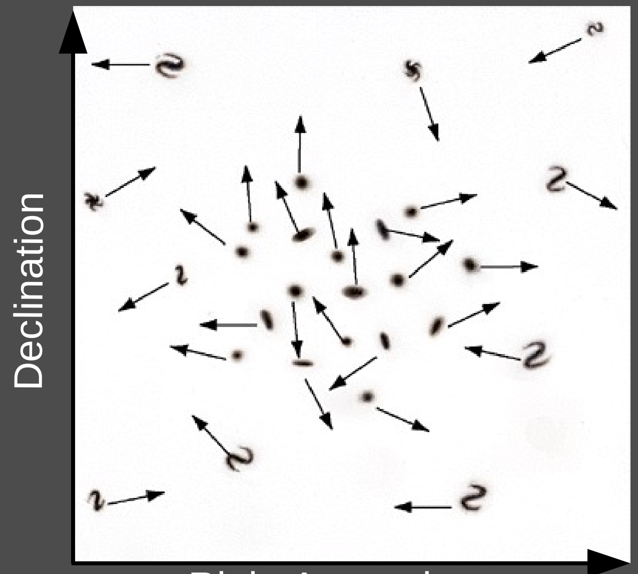
Galaxies

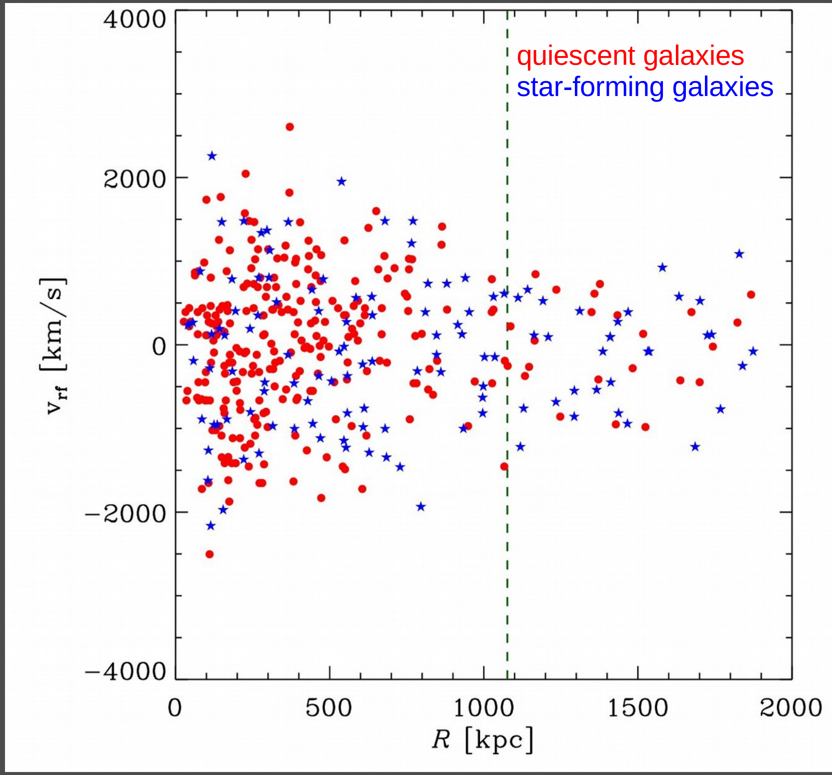
If also spectroscopic information is available, use **kinematics**:

Define the cluster center in RA, Dec (typically, the BCG, or the density center) and the projected radial distances from this center, ***R***

Define the cluster center in velocity, v_c (typically v_{BCG} or <v>) and the rest-frame line-of-sight velocities $\mathbf{v_{los}} \equiv \mathbf{v_{rf}} \equiv (v-v_c) / (1+v_c/c)$

correction for cosmological redshift broadening





AB et al. (2016): galaxies in the projected phase-space of z~1 clusters from the GCLASS

Masses & mass profiles

Galaxies

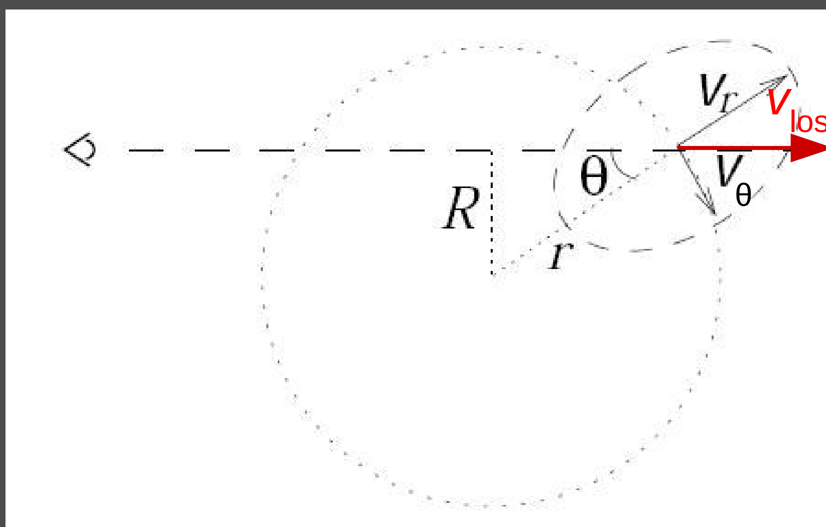
Use the projected phase-space distribution (R,v_{rf}) of cluster galaxies to recover the **intrinsic phase-space distribution f = f(E,L)** and from f(E,L) the gravitational potential and mass through the Poisson equation (e.g. *Wojtak et al. 2008*):

$$g(R, v_{\rm los}) = 2\int_R^{\infty} \frac{r\,dr}{(r^2 - R^2)^{1/2}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(E, L)\, dv_\theta\, dv_R$$

Energy, angular momentum

$$E = \Psi(r) - \frac{1}{2}\boldsymbol{v}^2$$

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{v}$$



$$\nabla^2 \Psi(\boldsymbol{r}) = -4\pi G \int f(\boldsymbol{r}, \boldsymbol{v})\, \mathrm{d}^3\boldsymbol{v}$$

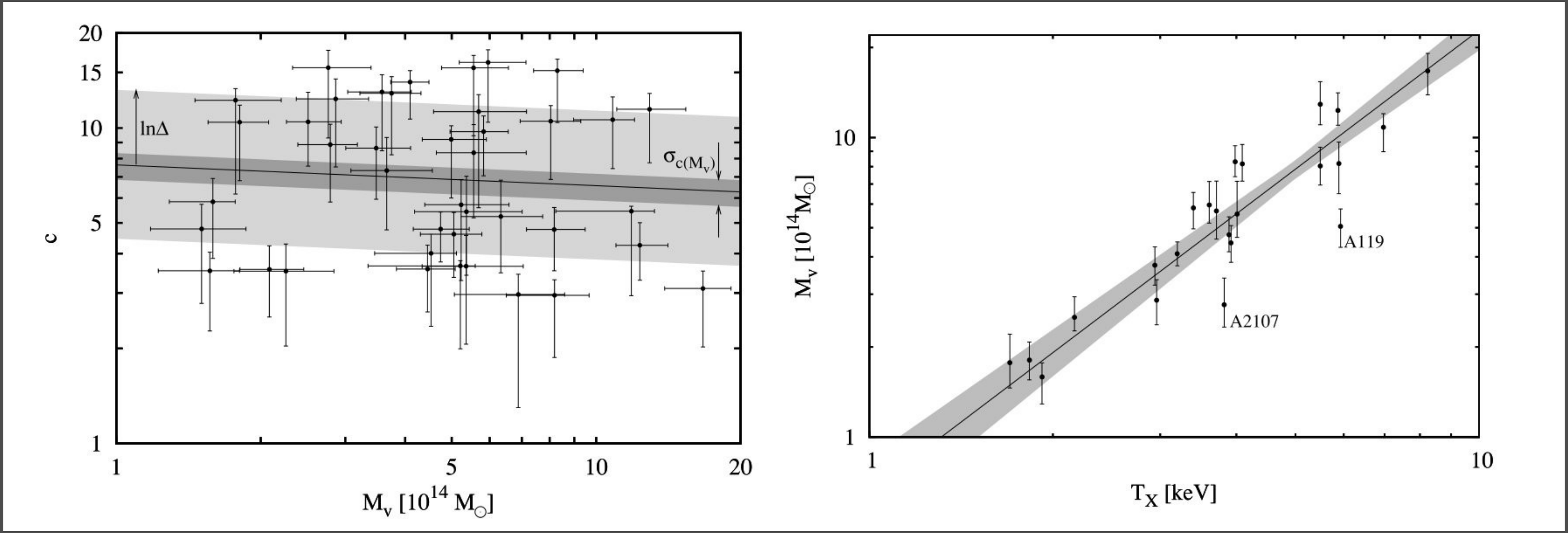
$$\Psi(r) = \int_r^{\infty} \frac{G M(r)\, \mathrm{d}r}{r^2}$$

The shape of halos f(E,L) is inferred from cosmological simulations

Masses & mass profiles

Galaxies

Some results on cluster M(r) from the f = f(E,L) method:



Wojtak & Łokas (2010): the concentration-mass relation of 44 nearby clusters and the correlation of the f(E,L)-derived mass with the T_X mass proxy

Masses & mass profiles

Galaxies

Solve the **Jeans equation** for a collisionless system of galaxies in dynamical equilbirum:

$$\frac{d(\nu\sigma_r^2)}{dr} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{d\Phi}{dr}$$

number density profile of cluster galaxies

gravitational potential

$$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

velocity anisotropy profile

velocity dispersion profile of cluster galaxies along the radial direction

$$\sigma_\theta^2 = \sigma_\phi^2$$

velocity dispersion profile along the tangential direction

Assuming the system is spherically symmetric, and in steady state, does not contract or expand, and does not rotate, and there is no preference for one of the two tangential components of the velocity dispersion.

Masses & mass profiles

Galaxies

$$\frac{d(\nu\sigma_r^2)}{dr} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{d\Phi}{dr}$$

dynamical pressure gradient

gravitational potential gradient

James Jeans

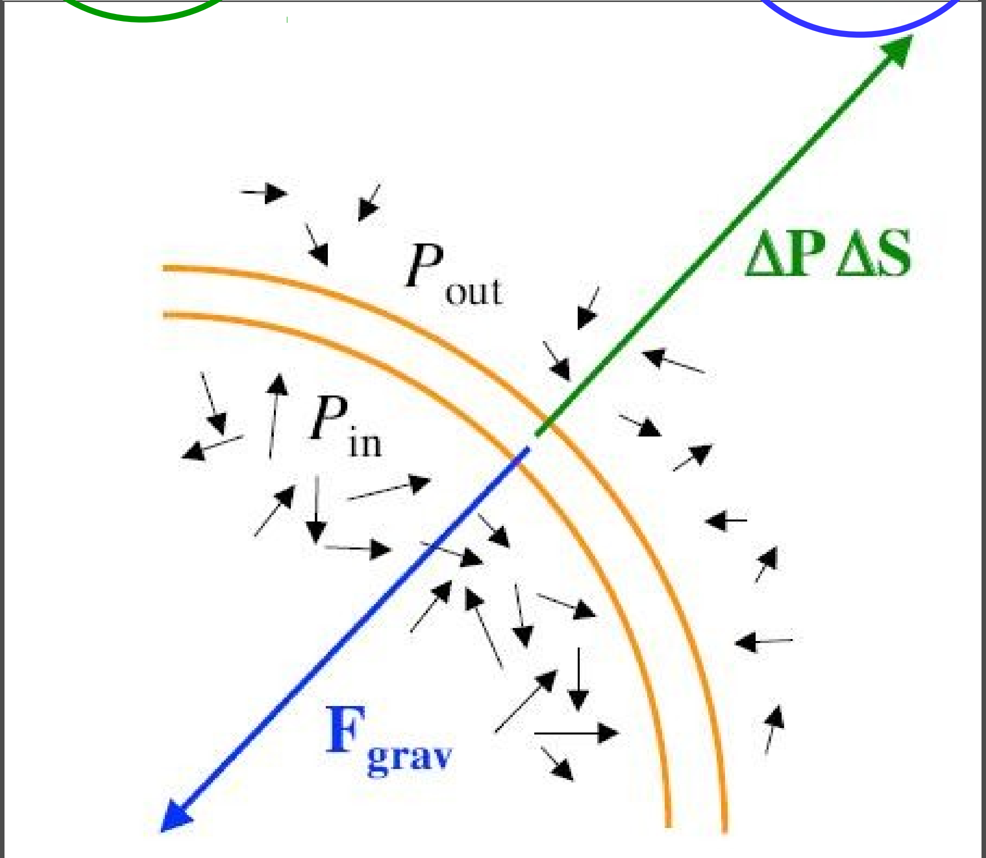


figure courtesy Gary Mamon

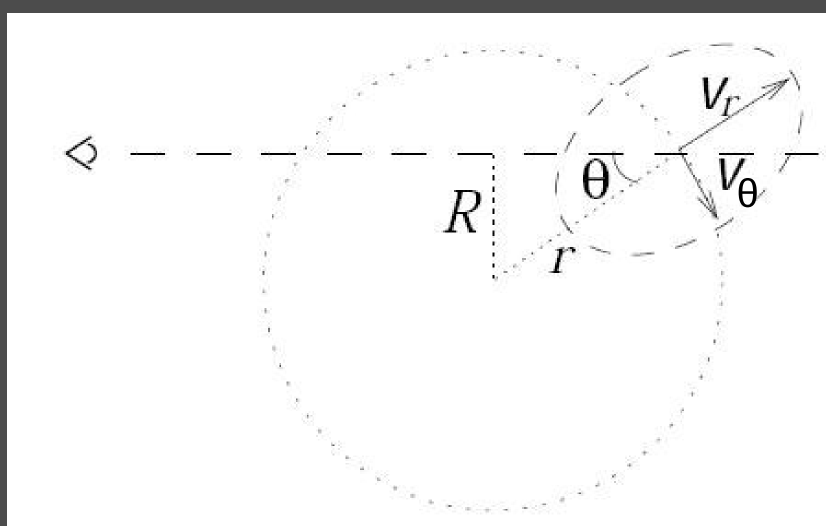
Masses & mass profiles

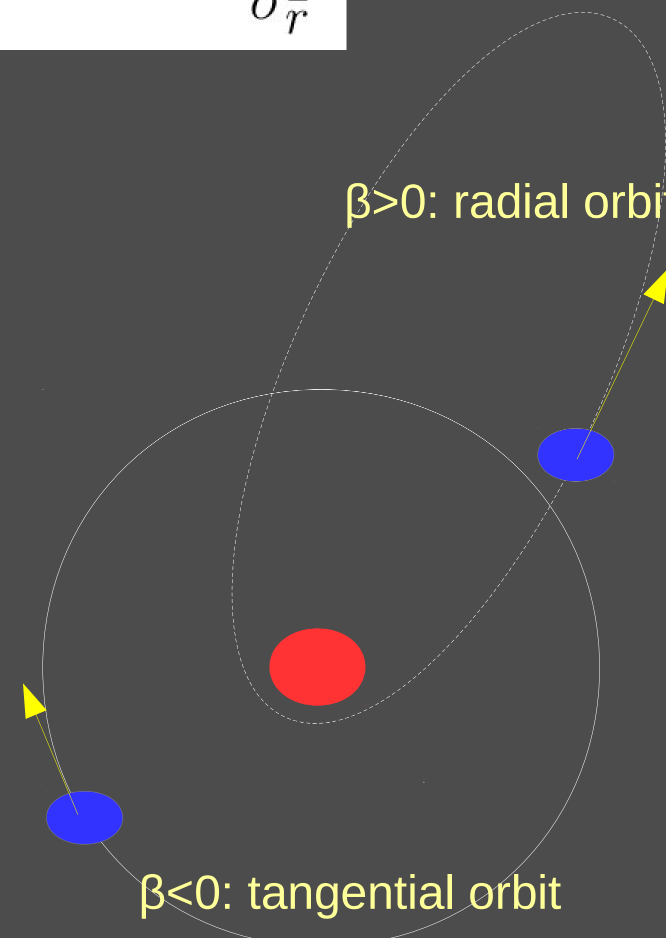
Galaxies

$$\frac{d(\nu\sigma_r^2)}{dr} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{d\Phi}{dr}$$

$$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

elongation of the velocity ellipsoid
→orbits of cluster galaxies





Masses & mass profiles

Galaxies

Given that $\frac{d\Phi}{dr} = \frac{GM(r)}{r^2}$, from the Jeans equation it is possible to derive **the system mass profile, M(r)**:

$$GM(r) = -r\sigma_r^2 \left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta \right)$$

Integrating the Jeans equation, and assuming the system is in steady state, we obtain the **scalar virial theorem:**

$$2K + W = 0$$

system's total kinetic energy

system's total potential energy

From the virial theorem it is possible to derive **the system total mass, M**:

$$GM = \frac{\sigma_{\rm tot}^2}{< b(r) r^{-1} >}$$

b(r) accounts for the possibility that the distribution of galaxies and the distribution of mass are different

Masses & mass profiles

Galaxies

The Jeans equation and the virial theorem use the **6** full phase-space coordinates.
However, for clusters, we only have observational access to 3 coordinates:
two spatial coordinates + the velocity along the line-of-sight (l.o.s.; from the redshift).

We need to **de-project** the two equations. For the **virial theorem:**

$$GM = \frac{\sigma^2_{\rm tot}}{< b(r) r^{-1} >} \quad \Longrightarrow \quad GM = \frac{3\pi\sigma^2_{\rm los}}{< R_{ij}^{-1} >}$$

harmonic mean radius

i,i = galaxy id. number

This is valid if we observe the entire (spherically symmetric) system, so that:

- $\sigma^2_{tot} = 3\ \sigma^2_{los}$ (since the velocity dispersion tensor has 3 components), independent of the shape of galaxy orbits (tangential vs. radial)
- $< b(r)\ r^{-1} > = < R_{ij}^{-1} > / \pi$, if we assume b(r) ≡ 1, the “light traces mass” hypothesis

Masses & mass profiles

Galaxies

In case we do not observe the entire system, the projected virial theorem needs to be corrected for the **surface pressure** term *(see The & White 1986)*:

$$GM = \frac{3\pi\sigma_{\rm los}^2}{< R_{ij}^{-1} >} - S[M(r), \beta(r), R_{\rm lim}]$$

where S is a function of the limiting radius of observation $R_{\rm lim}$, of the galaxy orbital distribution within the cluster, β(r), and (unfortunately) also of the mass distribution itself, M(r)

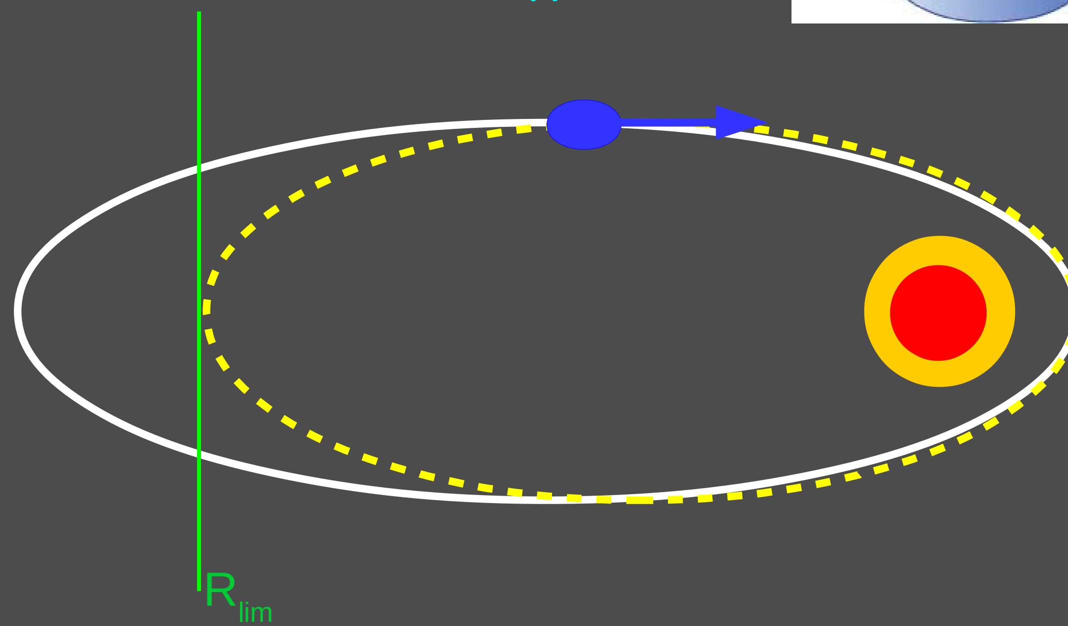
Blue ellipse: orbiting galaxy

Solid line: true orbit;

Dashed line: inferred orbit due to observational limitation;

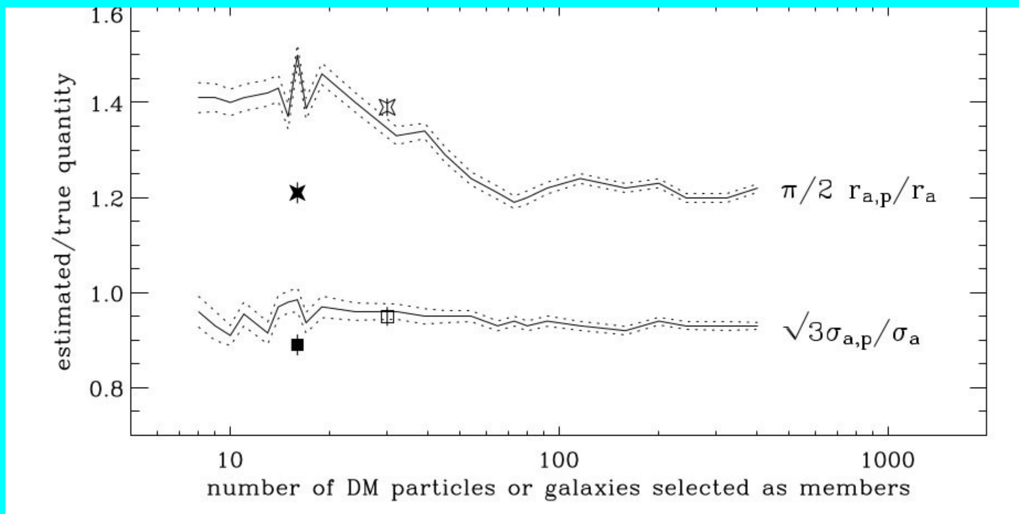
Red dot: true mass;

Orange dot: mass needed to keep the galaxy in the inferred orbital configuration



Masses & mass profiles

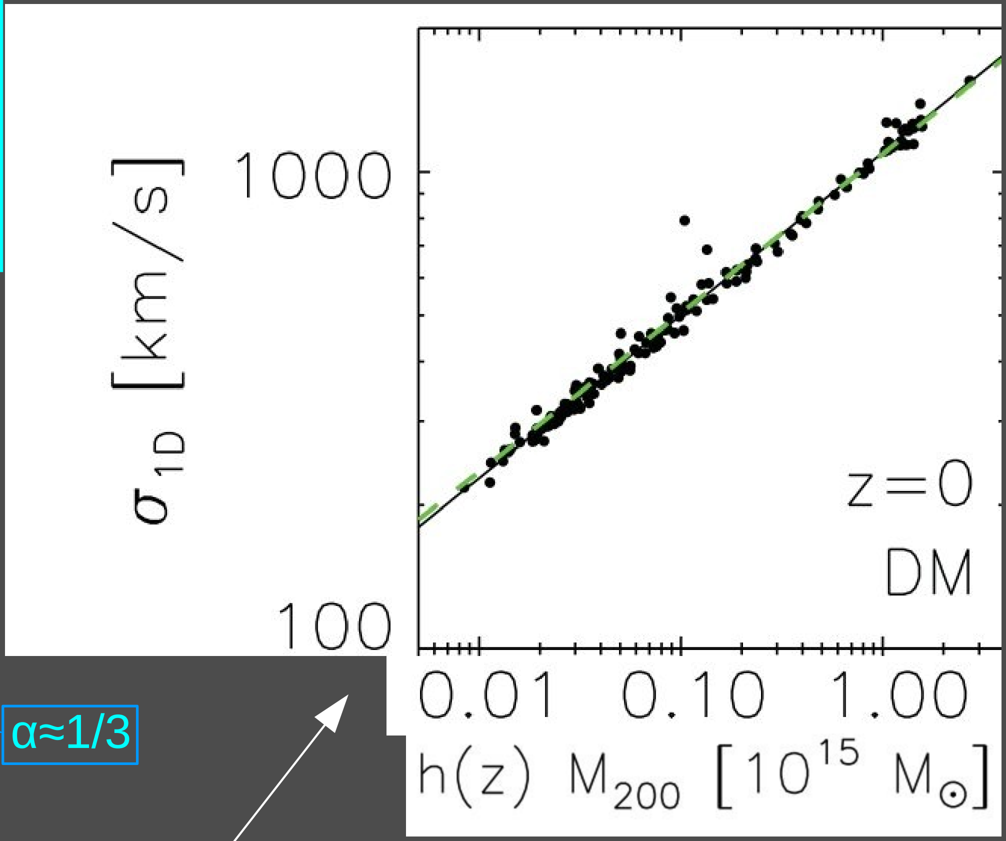
Galaxies



$$GM = \frac{3\pi\sigma_{\rm los}^2}{< R_{ij}^{-1} >}$$

AB et al. (2006):
the virial theorem tested on halos from numerical simulations

The estimate of $<R_{ij}^{-1}>$ is not robust when the number of cluster members is too low



One can **use σ_{los} as a mass proxy**, rather than calculating the mass from the virial theorem:

$$\frac{\sigma_{1\rm D}}{\rm km\,s^{-1}} = A_{1\rm D}\left[\frac{h(z)\,M_{200}}{10^{15}\,\rm M_\odot}\right]^\alpha$$

$\alpha \approx 1/3$

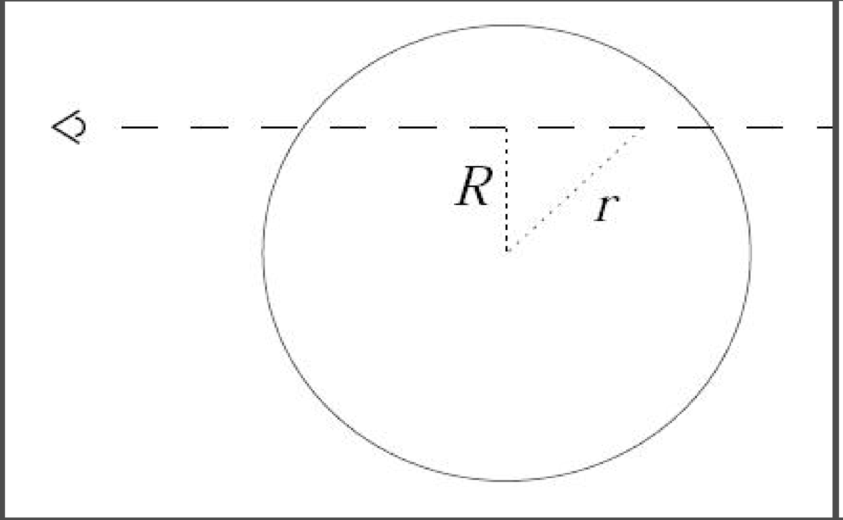
Simulations indicate an **intrinsic** scatter of ~0.06 dex at any redshift *(Munari, AB et al. 2013)*

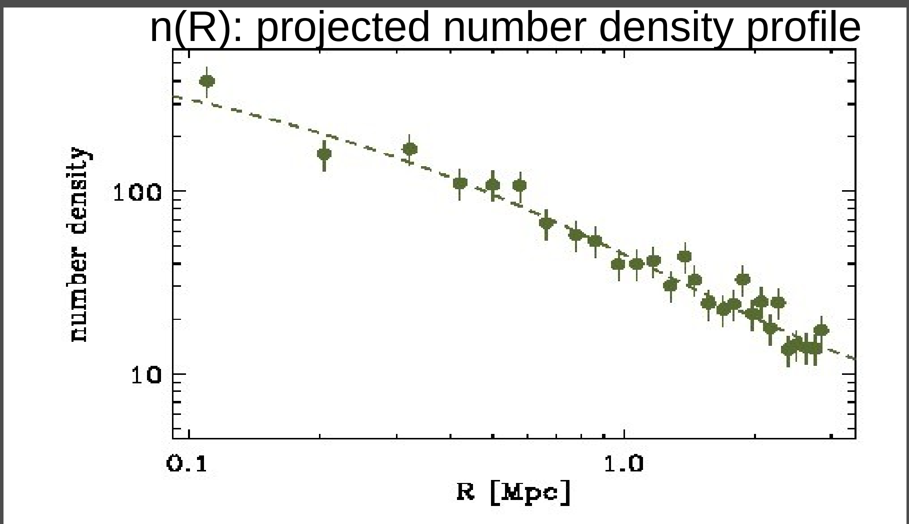
Masses & mass profiles

Galaxies

De-projecting the **Jeans equation**:

$$GM(r) = -r\sigma_r^2 \left(\frac{d \ln \nu}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + 2\beta \right)$$





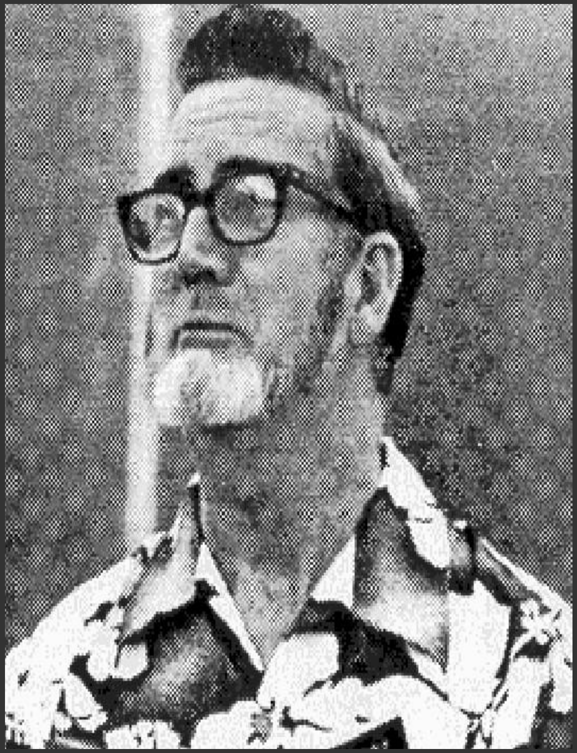
$$\nu(r) = -\frac{1}{\pi} \int_r^{\infty} \frac{dn}{dR} \frac{dR}{\sqrt{R^2 - r^2}}$$

Abel inversion equation

Masses & mass profiles

Galaxies

Niels Henrik Abel (∗1802 ✝1829) is not George Ogden Abell (∗1927 ✝1983)



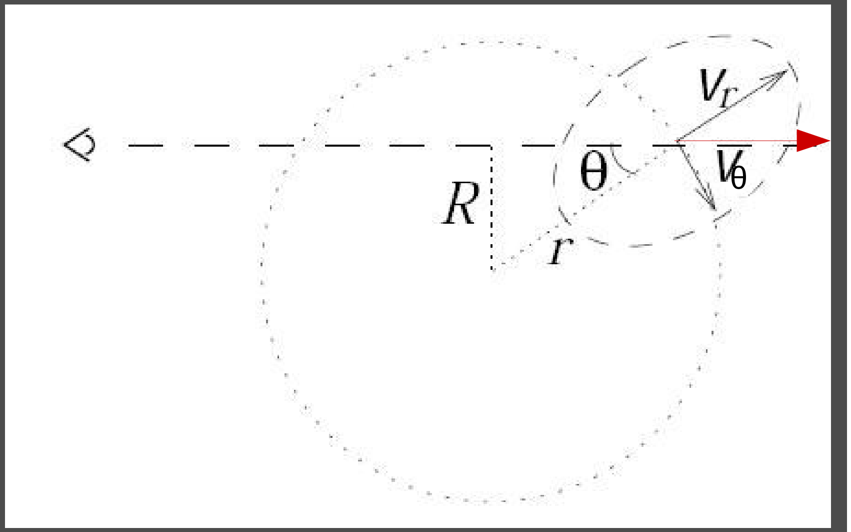
Masses & mass profiles

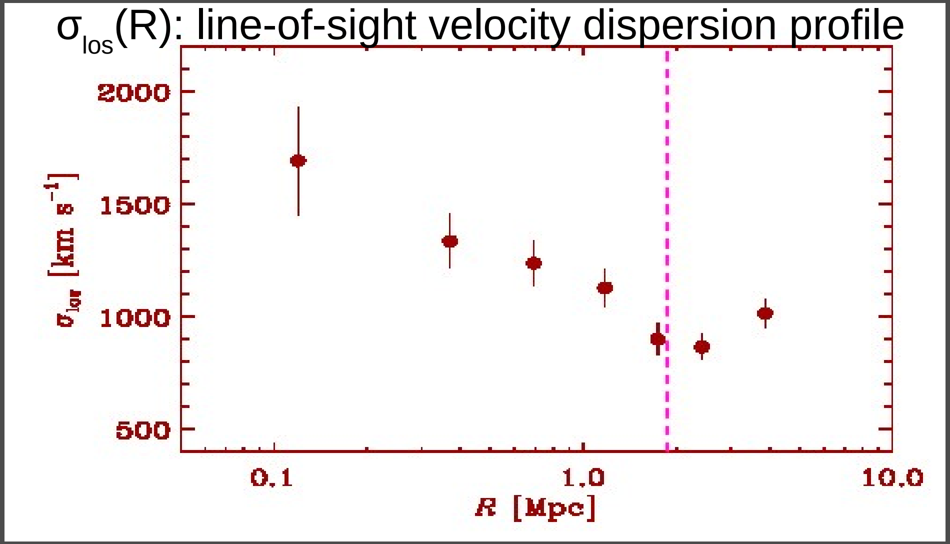
Galaxies

De-projecting the **Jeans equation**:

$$GM(r) = -r\sigma_r^2 \left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta \right)$$

$$\beta \equiv 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$





$$\sigma_r^2(r) = -\frac{1}{\pi\nu(r)} \int_r^\infty \frac{d(n\sigma_{\rm los}^2)}{dR} \frac{dR}{\sqrt{R^2 - r^2}}$$

Abel inversion equation, **valid ONLY for β(r)=0**

Masses & mass profiles

Galaxies

We observe:
1) the line-of-sight velocity dispersion profile $\boldsymbol{\sigma_{los}(R)}$

but to know **M(r)** we need
1) the radial velocity dispersion profile $\boldsymbol{\sigma_r(r)}$
2) and the velocity anisotropy profile **β(r)** – or, equivalently, $\sigma_\theta(r)$

The solution to the Jeans equation is degenerate between M(r) and β(r)

(This is also true for the total mass from the virial theorem, unless we assume to know the mass distribution)

How do we solve this
“mass-orbit degeneracy”?

Masses & mass profiles

Galaxies

How we solve the **“mass-orbit degeneracy”**:

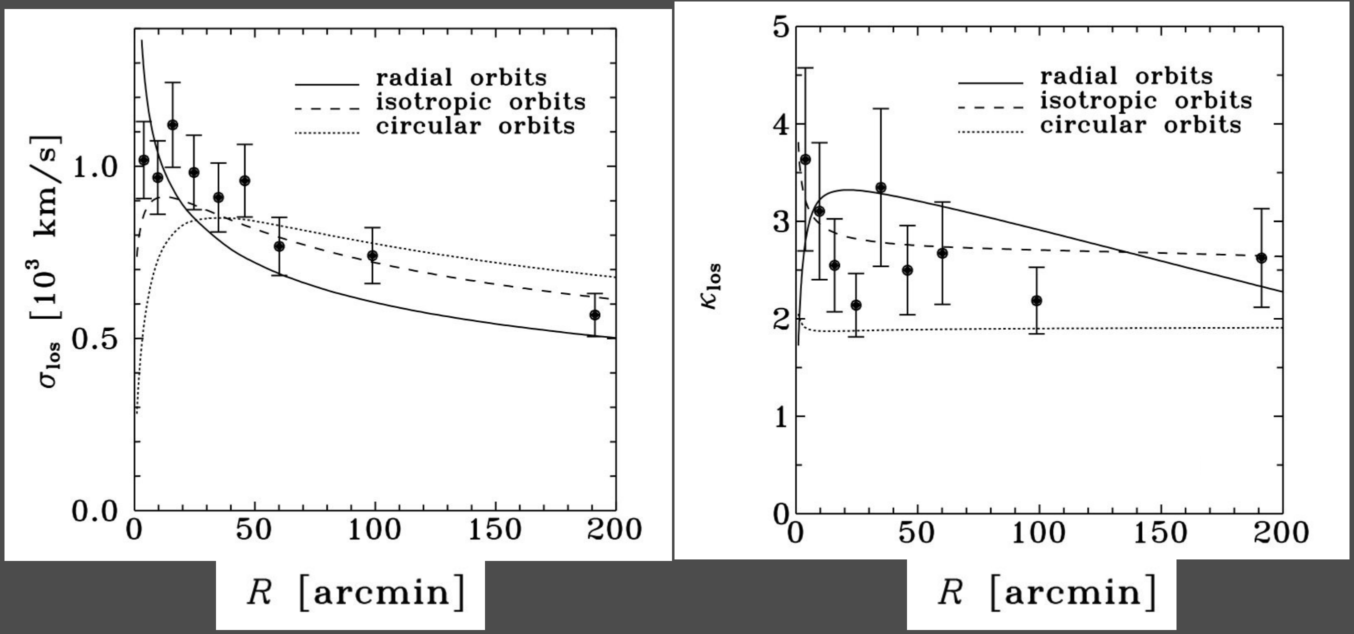
Several possibilities:

➔ Trust cosmological numerical simulations and use their β(r)

➔ Solve multiple Jeans/Virial equations separately for ≠ tracers (e.g. ellipticals/spirals)
– this works if they have ≠ β(r), since M(r) is unique *(AB+Poggianti 2009)*

➔ Go beyond the Jeans equation, considering higher moments of the velocity distribution
– e.g. the velocity kurtosis profile in addition to the velocity dispersion profile
(Łokas & Mamon 2003)

Łokas & Mamon (2003)’s analysis of the Coma cluster; different orbital shapes – i.e. β(r) – can be distinguished by the shapes of the los velocity dispersion, $\sigma_{los}(R)$, and los kurtosis, $k_{los}(R)$, profiles

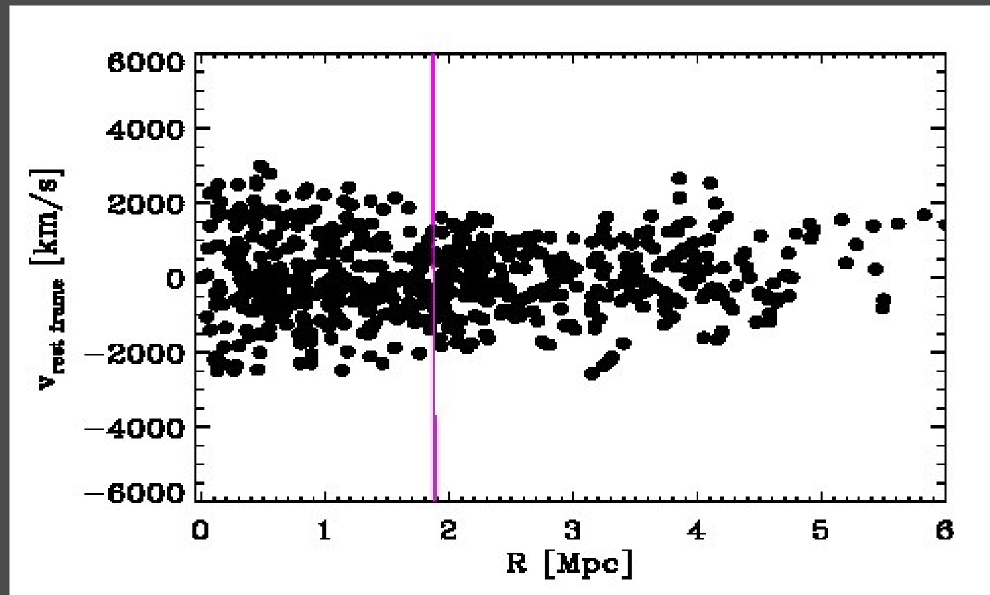


Masses & mass profiles

Galaxies

MAMPOSSt

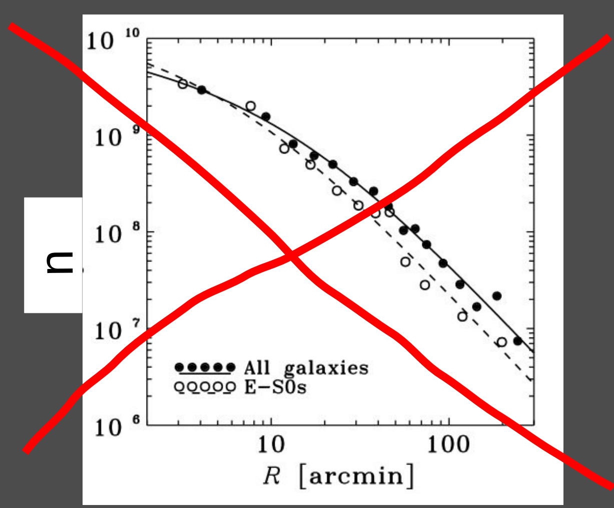
direct maximum likelihood fit to the phase-space distribution of cluster galaxies in projection

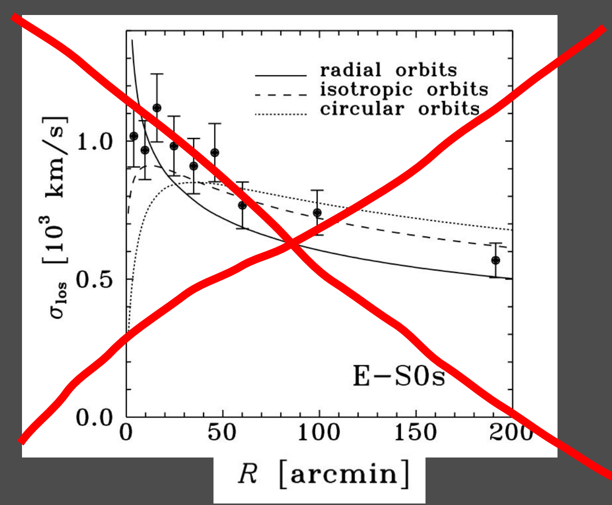


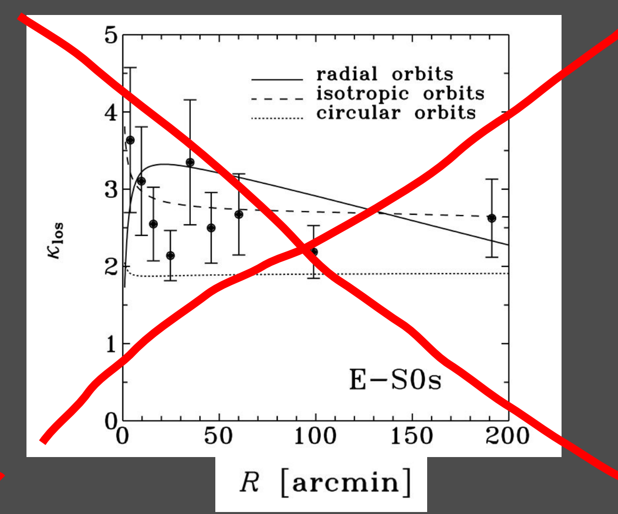
Modelling **A**nisotropy and **M**ass **P**rofiles of **O**bserved **S**pherical **S**ystems

[Mamon, AB, Boué 2013]

Does **not** fit the projected number density, velocity dispersion, kurtosis profiles n(R) , σ_{los}(R), k_{los}(R):





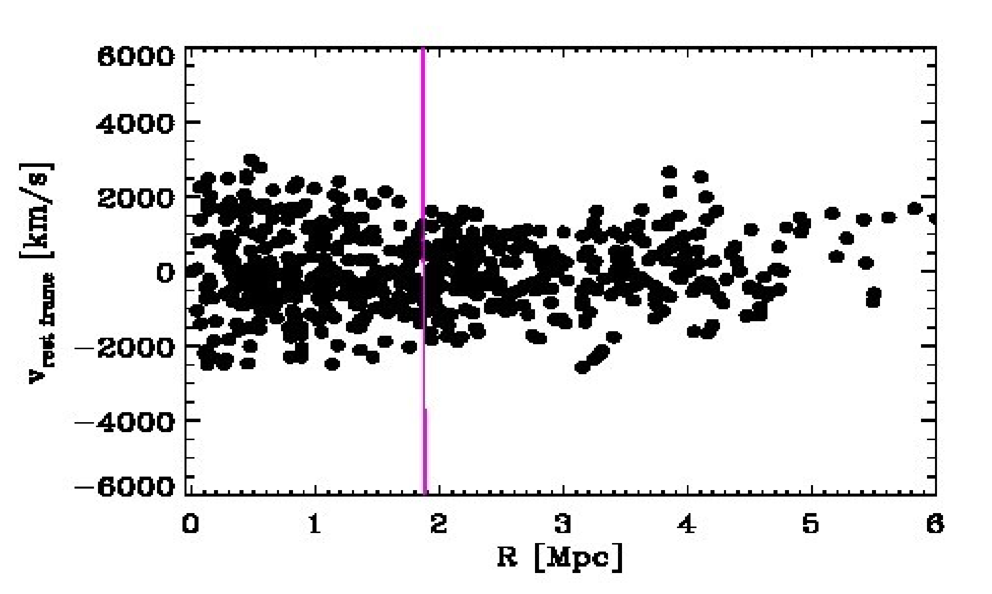


Masses & mass profiles

Galaxies

MAMPOSSt

direct maximum likelihood fit to the phase-space distribution of cluster galaxies in projection



Modelling **A**nisotropy and **M**ass **P**rofiles of **O**bserved **S**pherical **S**ys**t**ems

[Mamon, AB, Boué 2013]

Computes the probability p_i of observing a galaxy i at a projected radial distance R_i from the cluster center with a rest-frame line-of-sight velocity v_i, given models for:

- ➔ the 3D number density profile ν(r,**κ**)
- ➔ the mass profile M(r,**λ**)
- ➔ the velocity anisotropy profile β(r,**μ**)

Find the optimal (best-fit) parameters **κ, λ, μ** by maximizing: $\prod_{i=1}^{N} p_i$

Masses & mass profiles

Galaxies

The surface density of observed objects in projected phase space is:

$$g(R, v_z) = \text{n}(R)\ \langle h(v_z|R, r)\rangle_{\text{LOS}}$$
$$= 2\int_R^{\infty} \frac{r\,\nu(r)}{\sqrt{r^2 - R^2}}\, h(v_z|R, r)\, \mathrm{d}r\,, \qquad (4)$$
$$= 2\int_R^{\infty} \frac{r\,\mathrm{d}r}{\sqrt{r^2 - R^2}} \int_{-\infty}^{+\infty} \mathrm{d}v_\perp \int_{-\infty}^{+\infty} f(r, v_z, v_\perp, v_\phi)\, \mathrm{d}v_\phi, \qquad (5)$$

The MAMPOSSt equations

Hence, the probability density of observing an object at position (R, v_z) is:

$$q(R, v_z) = \frac{2\pi\, R\, g(R, v_z)}{\Delta N_{\mathrm{p}}}$$
$$= \frac{4\pi\, R}{\Delta N_{\mathrm{p}}} \int_R^{\infty} \frac{r\,\nu(r)}{\sqrt{r^2 - R^2}}\, h(v_z|R, r)\, \mathrm{d}r$$

That can be solved by assuming a distribution for 3D galaxy velocities (e.g. Gaussian):

$$h(v_z|R, r) = \frac{1}{\sqrt{2\pi\sigma_z^2(R, r)}}\, \exp\left[-\frac{v_z^2}{2\,\sigma_z^2(R, r)}\right]$$

$$\sigma_z^2(R, r) = \left[1 - \beta(r)\left(\frac{R}{r}\right)^2\right]\sigma_r^2(r).$$

where $\sigma_r^2(r)$ is obtained from the Jeans equation, given M(r) and β(r)

$$\sigma_r^2(r) = \frac{1}{\nu(r)} \int_r^{\infty} \exp\left[2\int_r^s \beta(t)\frac{\mathrm{d}t}{t}\right] \nu(s) \frac{G M(s)}{s^2}\, \mathrm{d}s$$

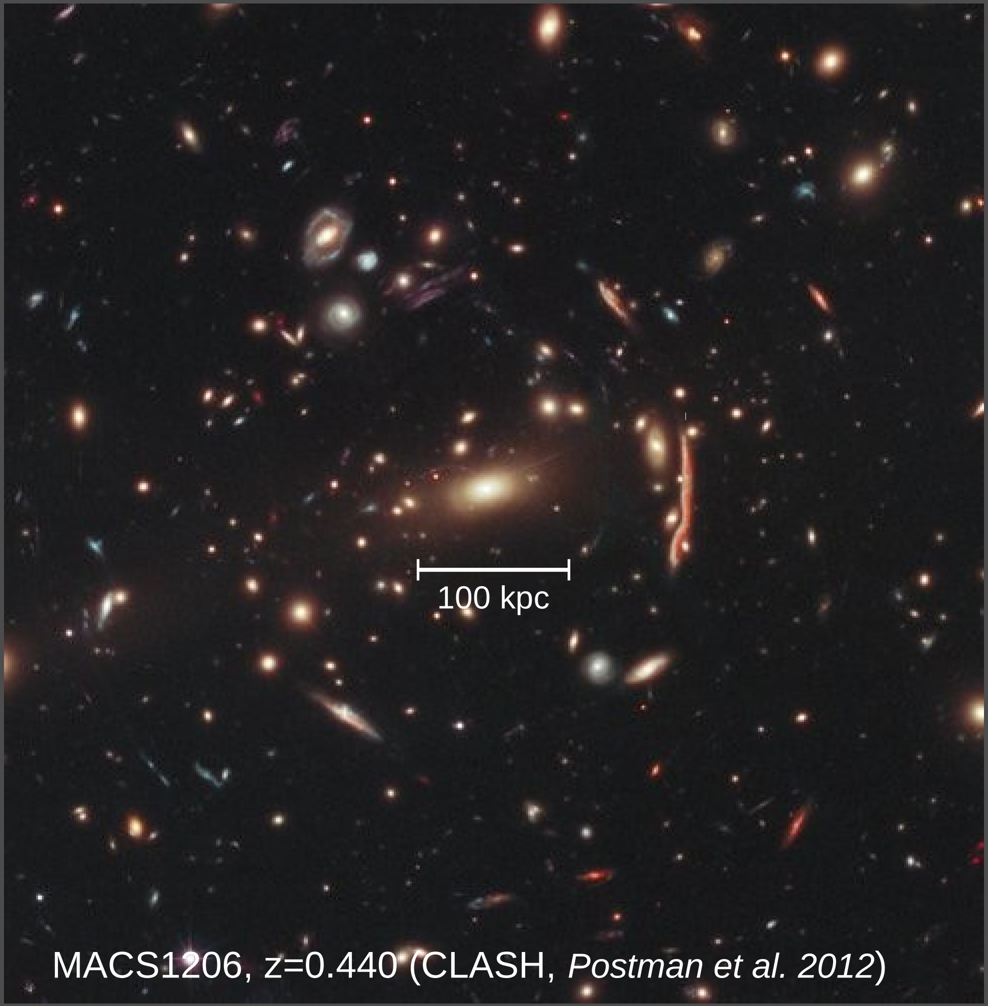
Masses & mass profiles

Galaxies

Methods described so far use galaxies as point-like tracers of the cluster potential.

Many clusters have a central BCG extending to ≥ 50 kpc

How can we probe the cluster gravitational potential in the very center (r ≤ 50 kpc)?



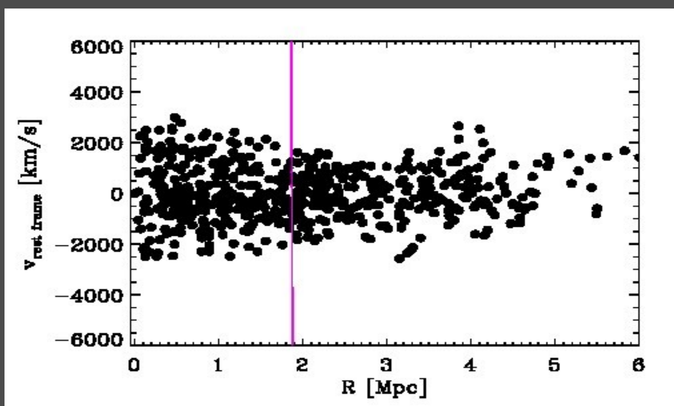
MACS1206, z=0.440 (CLASH, *Postman et al. 2012*)

Masses & mass profiles

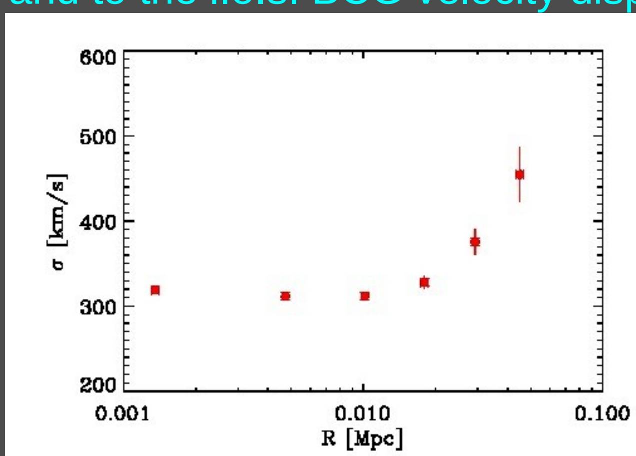
Galaxies

How we can probe the cluster gravitational potential in the very center:
use the **BCG stellar kinematics** to probe the cluster gravitational potential

Joint Maximum Likelihood fit to the projected phase-space distribution of cluster members:



and to the l.o.s. BCG velocity dispersion profile:



Constrain the best-fit parameters of the cluster mass profile M(r) parameterized as a sum of:

DM mass profile
BCG stellar mass profile
Intra-Cluster gas mass profile
stellar mass profile of all other galaxies

BCG stellar mass

DM

Intra-cluster gas

$$M(r) = M_{\rm gNFW} + M_{\rm Jaffe} + M_{\rm ICM} + M_{\rm gal}$$

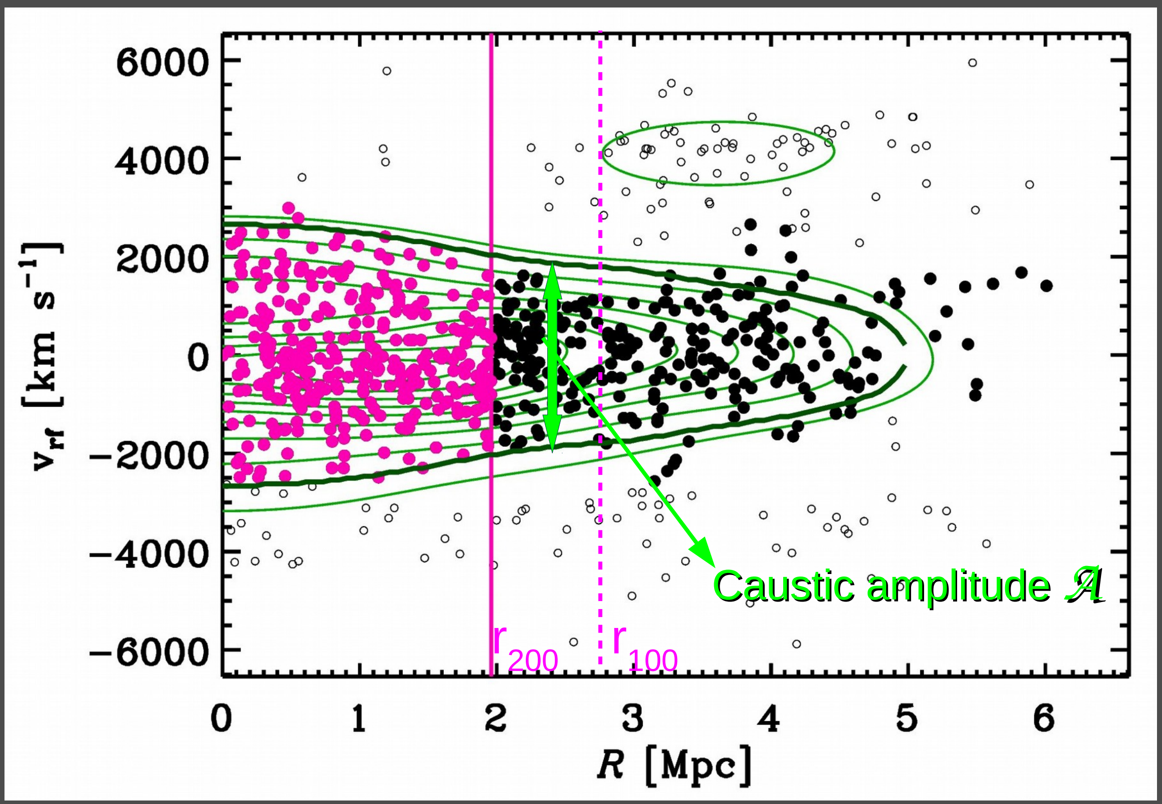
galaxies stellar mass

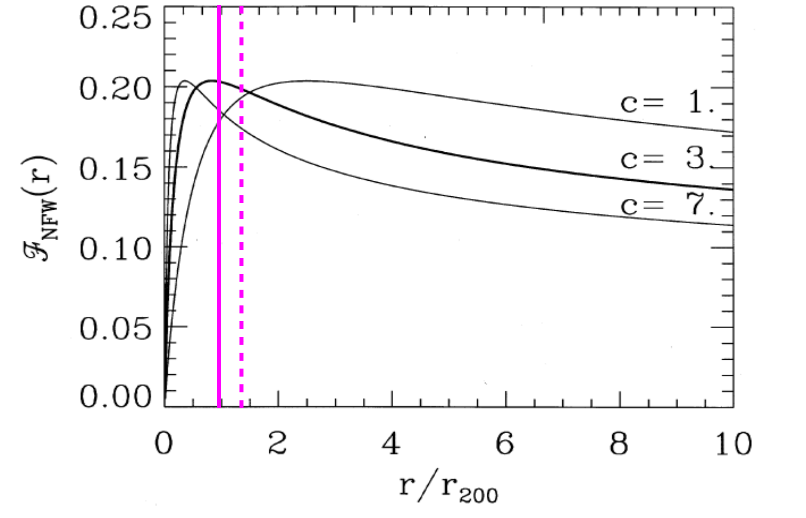
Masses & mass profiles

Galaxies

Dynamical equilibrium is expected within the "virial radius", i.e. ≈ r_{100} at z~0.
At larger radii, the Jeans equation may not apply.
At larger radii, use the **Caustic** technique *(Diaferio & Geller 1997; Diaferio 1999)*:

$$GM(<r) - GM(<r_0) = \int_{r_0}^{r} \mathcal{A}^2(x)\mathcal{F}_\beta(x)\,\mathrm{d}x$$



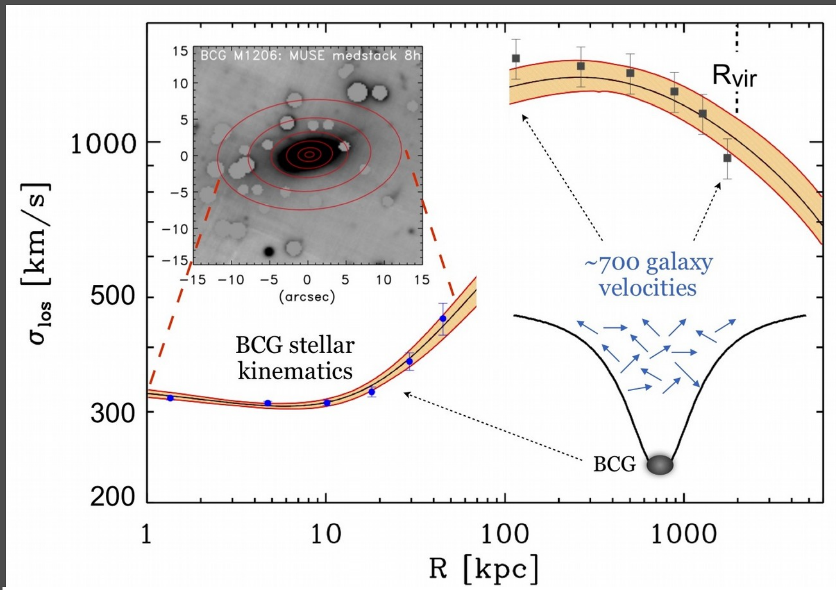


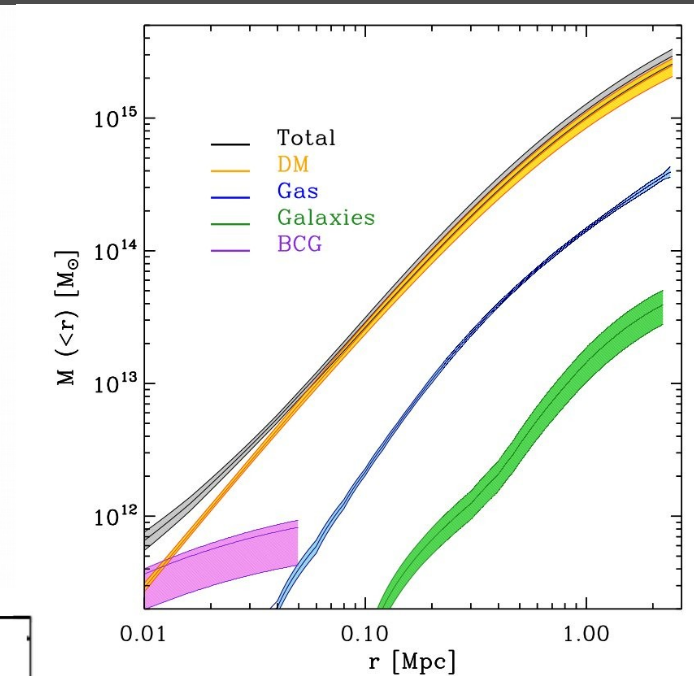
$\mathcal{F}(r)$ depends on M(r) itself within the virial region but it is ≈ constant outside →can solve the integral on the r.h.s.

Use the Jeans equation *(MAMPOSSt)* in the virial region to determine $M(<r_{200})$ or $M(<r_{100})$ and the Caustic technique outside this radius *(AB+Girardi 2003)*

Masses & mass profiles

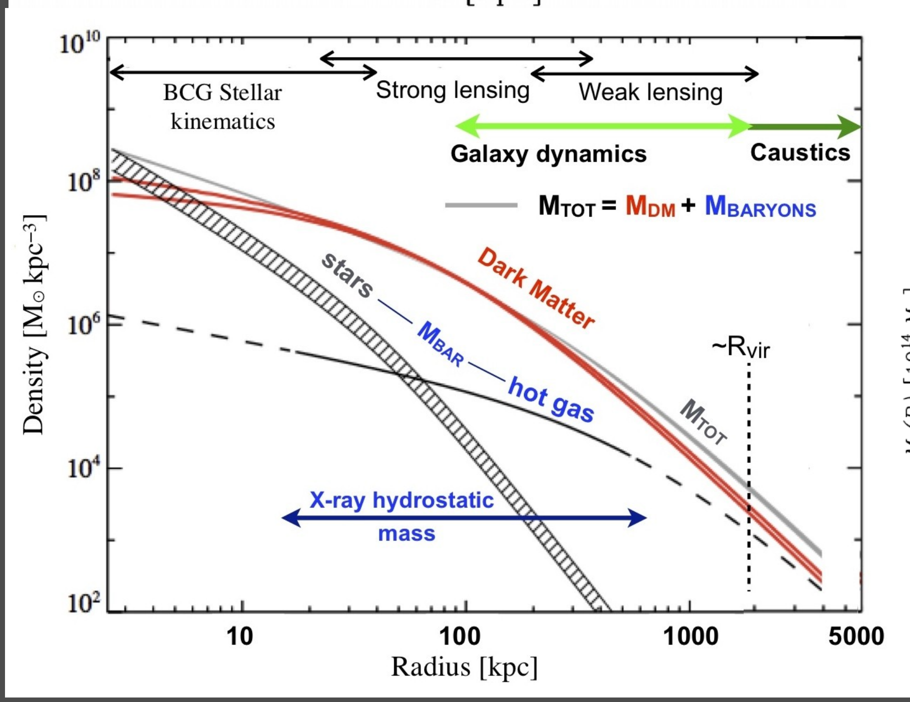
Galaxies

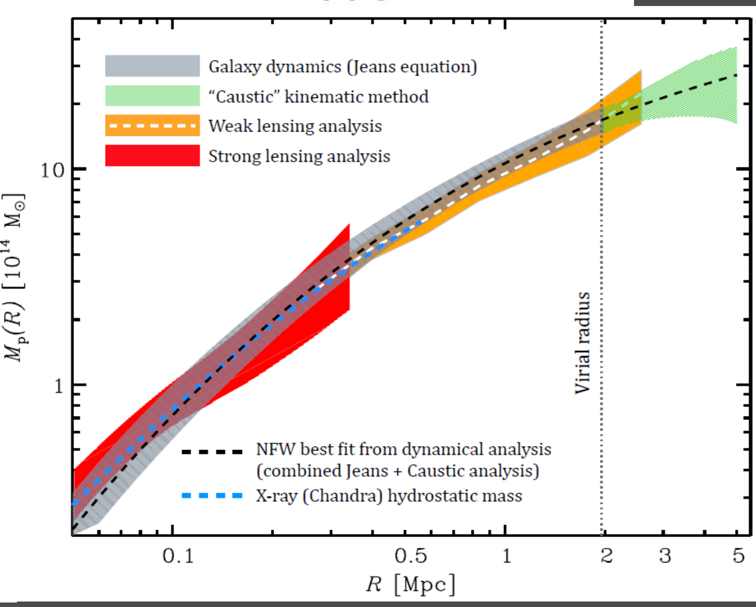




Sartoris, AB et al. (2020), AB et al. (2013) and courtesy P. Rosati:

The mass profiles of two clusters of galaxies (and its components) from 10 to >2000 kpc, as inferred from kinematics and compared with X-ray and lensing determinations.





Masses & mass profiles

Galaxies

Observational problems:

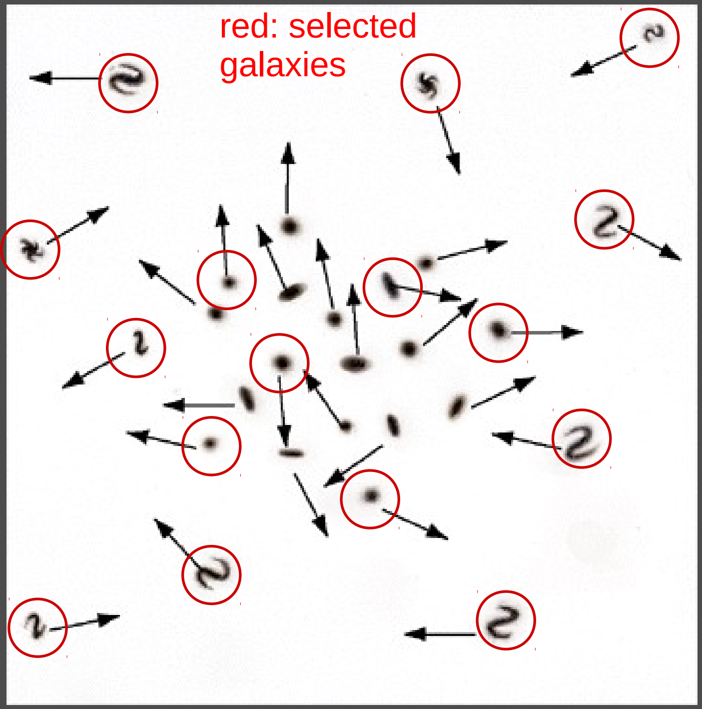
- ➔ incompleteness
- ➔ collisional processes
- ➔ deviation from dynamical equilibrium
- ➔ interlopers
- ➔ triaxiality
- ➔ poor statistics

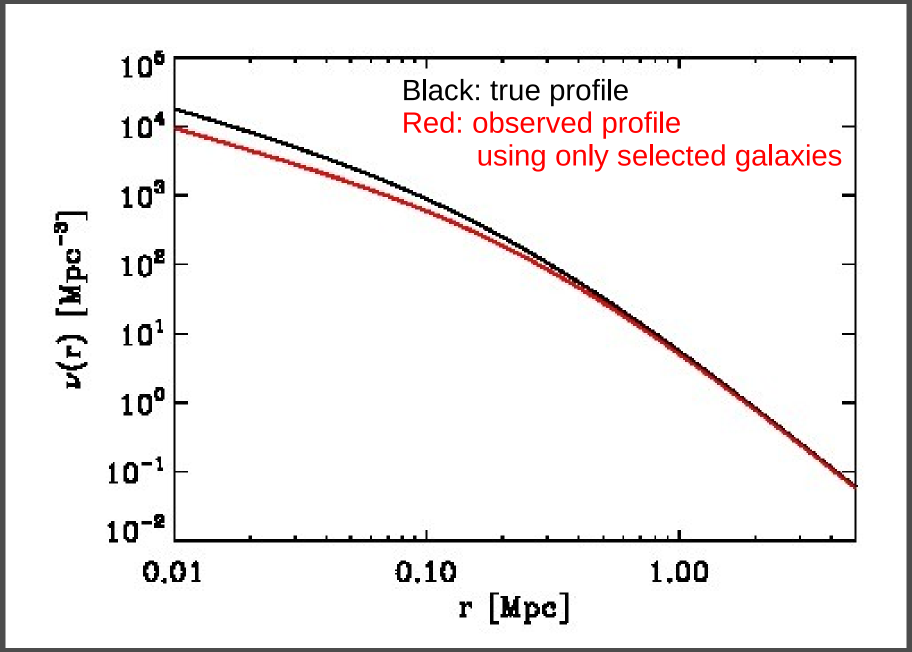
Masses & mass profiles

Galaxies

The **incompleteness** of the spectroscopic sample can affect estimates related to the spatial distribution of galaxies, such as:

- the harmonic mean radius $< R_{ij}^{-1} >$ (virial theorem), and
- the number density profile (Jeans equation)





Solutions:

- ✓ Estimate the incompleteness and correct the spectroscopic sample
- ✓ Use a substitute sample that is complete (e.g. photometric sample)

Masses & mass profiles

Galaxies

Does the **incompleteness** of the spectroscopic sample also affect estimates of the cluster velocity distribution?

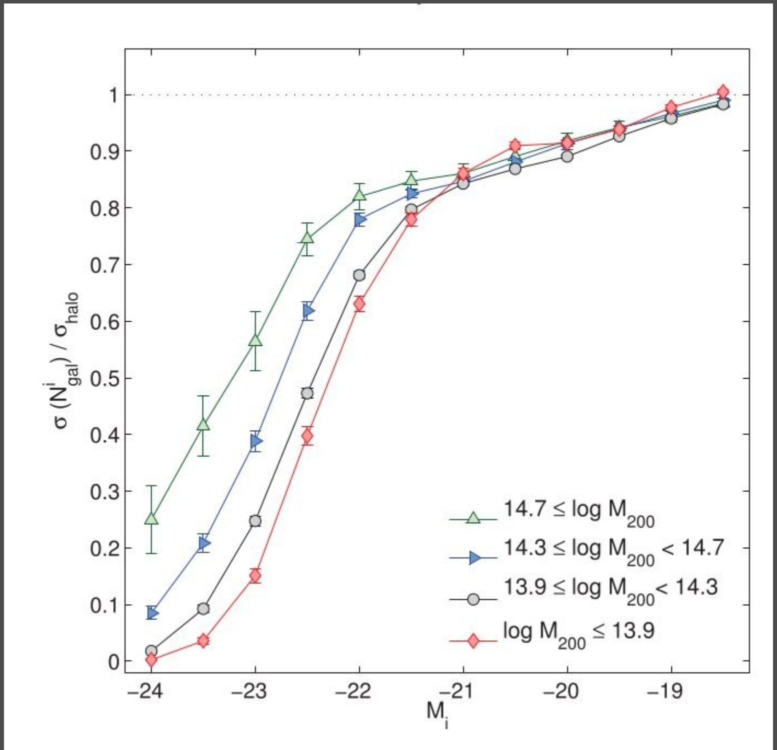
Masses & mass profiles

Galaxies

The **incompleteness** of the spectroscopic sample does **not** affect estimates of the cluster velocity distribution (or only mildly so), because:

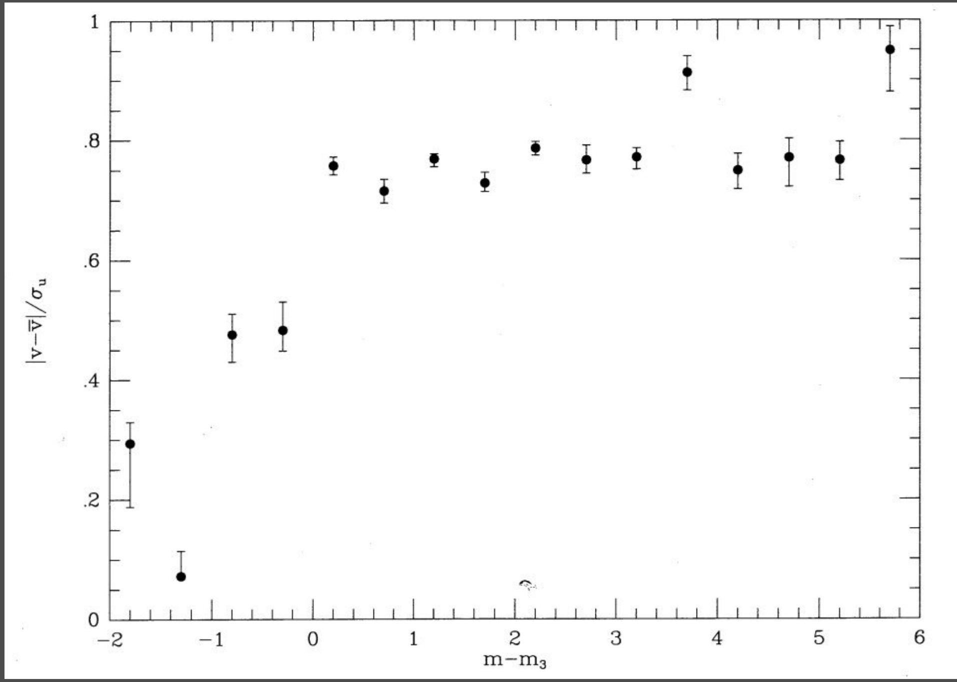
a) it is impossible to observationally pre-select cluster galaxies based on their v_{rf}

b) cluster member v_{rf} are only mildly correlated with their positions



Old et al. (2013): estimate of σ_v for clusters from numerical simulations

However, some effect can be present when the selection is not random but based on galaxy magnitude, because of **dynamical friction**:

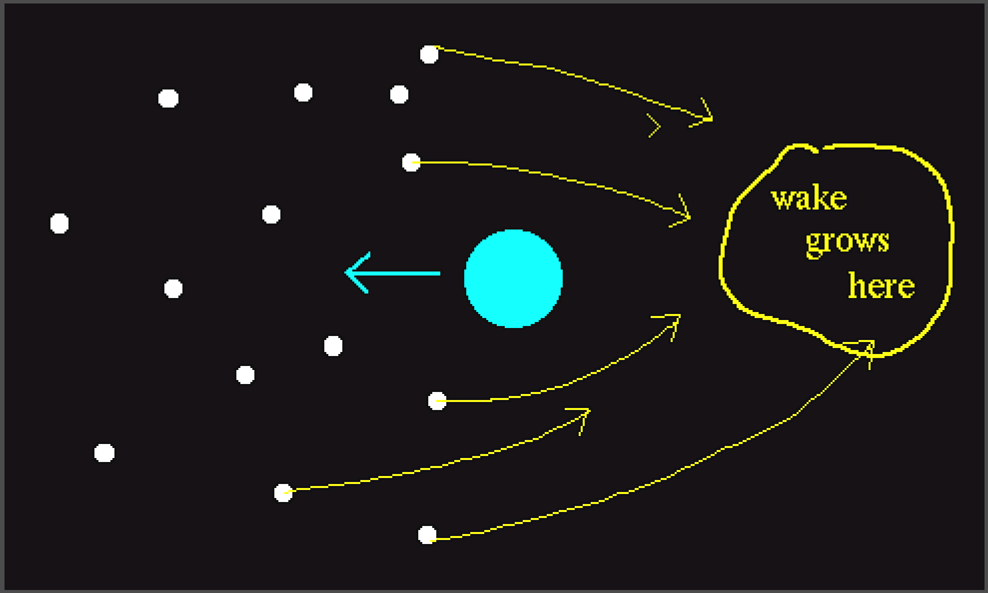


AB et al. (1992): estimate of the amplitude of the velocity scale for observed clusters

Masses & mass profiles

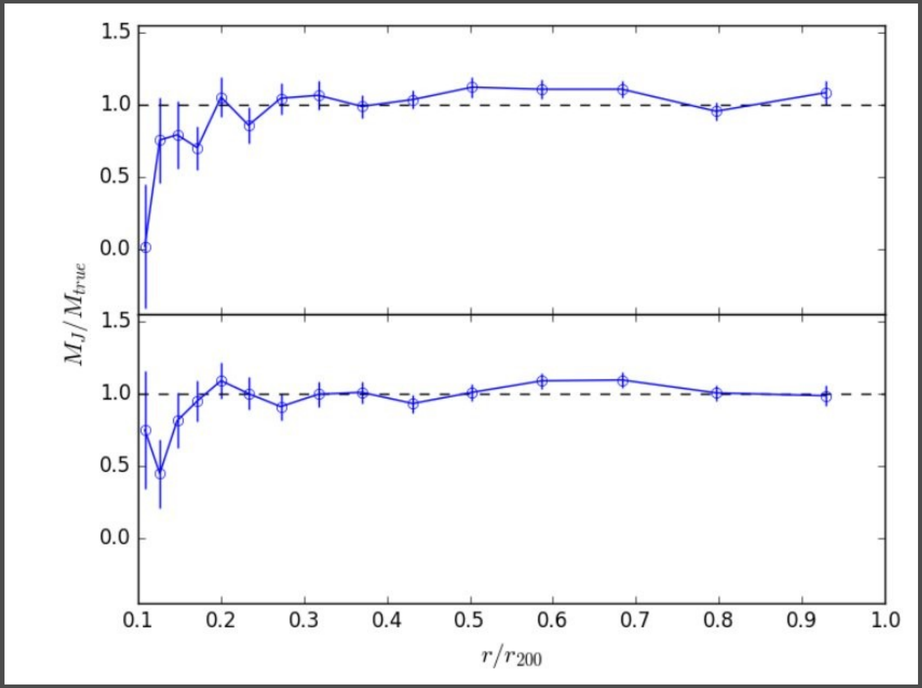
Galaxies

Dynamical friction is the only relevant collisional process that can hamper the use of the Jeans equation:



It is more relevant for more massive galaxies in denser regions, and tends to reduce their speed: $t_{df} \propto v_g^3/(m_g\ \rho)$

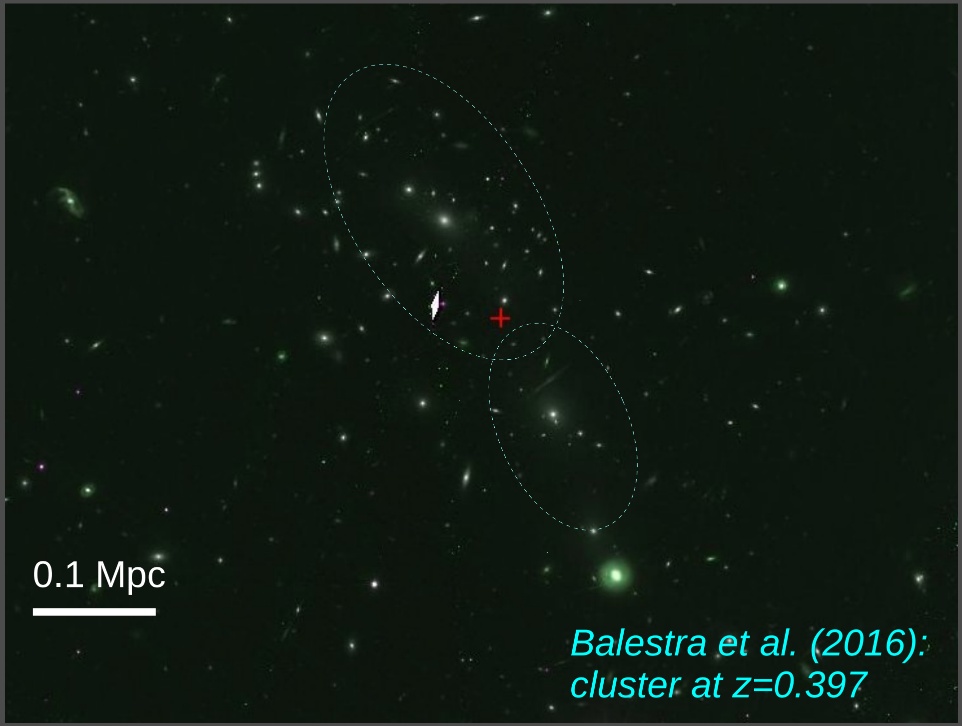
Tagliaferro, AB et al. (2021): Jeans analysis of cluster dynamics using two semi-analytical numerical simulations shows the effect of dynamical friction on the mass estimate

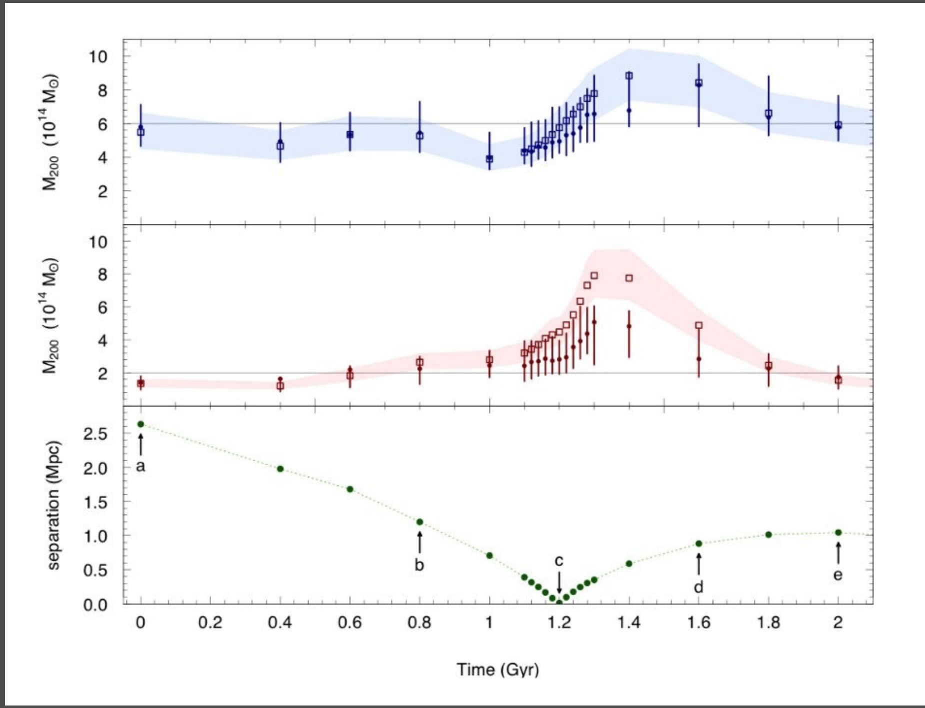


Masses & mass profiles

Galaxies

Clusters are young cosmic objects, they are still forming by accretion of galaxies and smaller groups of galaxies from the surrounding field. **Deviation from dynamical relaxation** can occur and it affects mass estimates





Estimated mass from σ_{los} (squares) and Caustic (dots) of a simulated cluster (blue) and its colliding subcluster (red) vs. time during a merger *(Monteiro-Oliveira et al. 21)*

(Partial) solution:

Identify substructures (i.e. colliding groups) and remove them from the sample used for the dynamical estimate

Masses & mass profiles

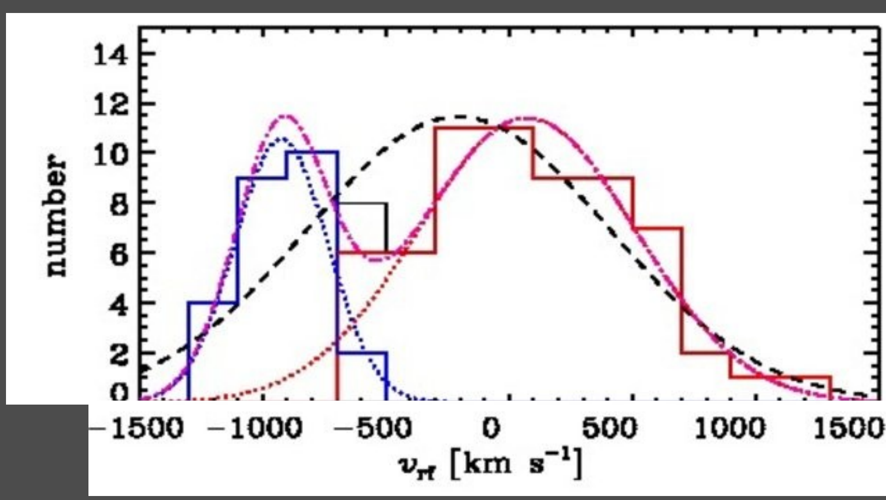
Galaxies

Identification of substructures: the most popular techniques are:

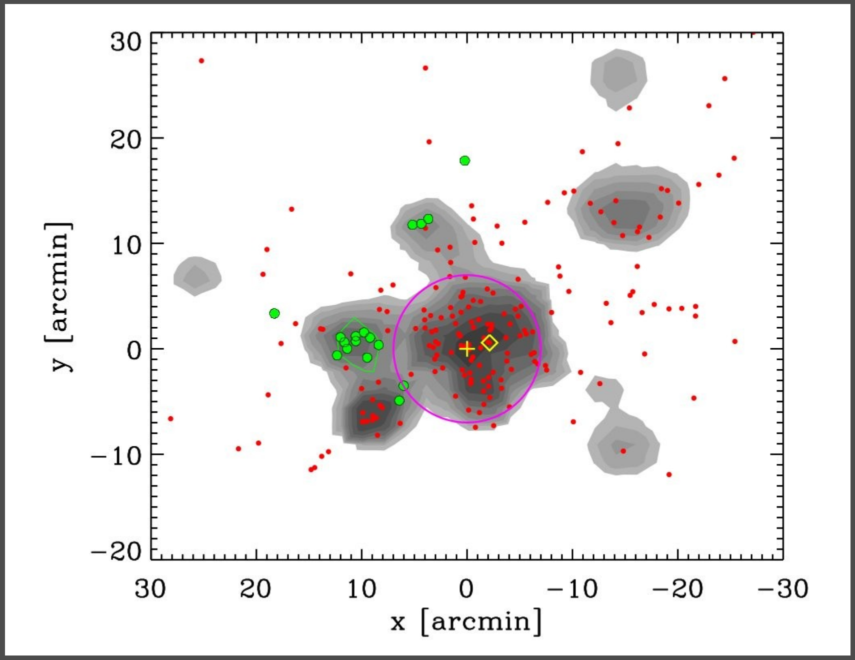
- Kernel Mixture Model (KMM, *Ashman et al. 1994*)
- *Dressler & Shectman (1988)*, now upgraded to DS+ (*AB et al. 2021*)

KMM estimates the probability that the velocity distribution is better represented by a mixture of k Gaussians rather than a single one

DS+ considers all possible groups of any multiplicity of neighboring galaxies in position, and flag as subclusters those groups whose velocity distribution differ from that of the cluster as a whole



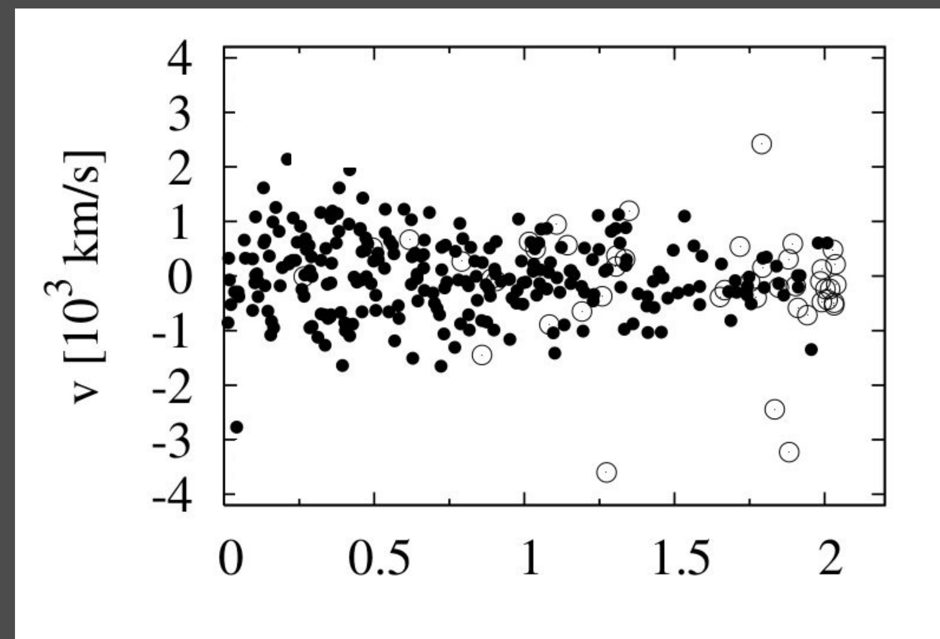
AB et al. (2017): Abell 315; KMM identifies 2 Gaussians and DS+ identified 2 groups (green)



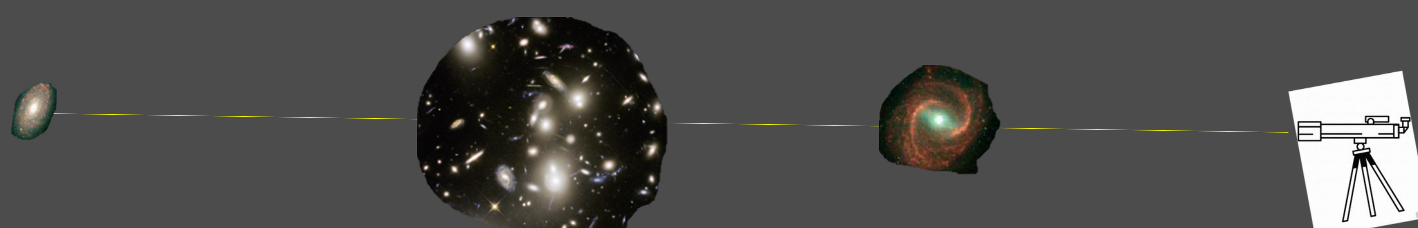
Masses & mass profiles

Galaxies

Even if we do not suffer from incompleteness and the cluster is dynamically relaxed, we must identify cluster members ≡ galaxies with $r < k\, r_\Delta$, and/or gravitationally bound to the cluster, and clean the observational sample from **interlopers** ≡ galaxies erroneously identified as members because of projection effects



Projected phase-space distribution of a simulated cluster (*Wojtak et al. 2008).* Filled (open) dots: members (interlopers)

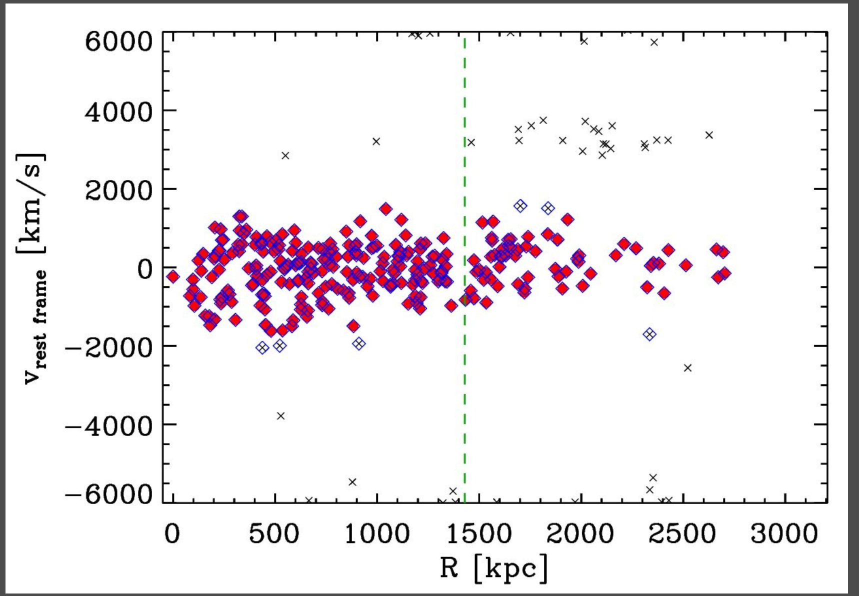


Masses & mass profiles

Galaxies

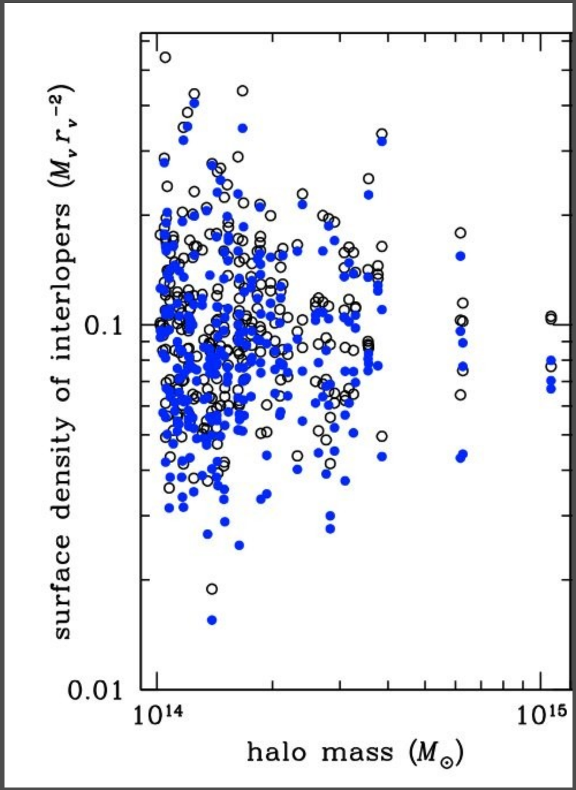
Interloper removal methods:

Direct: based on the location of a galaxy in projected phase-space (PPS), can use theoretical models to define the cluster boundary in PPS (e.g. "Clean" by *Mamon, AB, Boué 2013*) or search for gaps in PPS to separate the region occupied by members from that occupied by interlopers (e.g. "Shifting Gapper" by *Fadda et al. 1996;* "CLUMPS" by *AB et al. 2021*). These techniques can also be combined.



Cluster Abell 2457 from the ΩWINGS survey:
x = interlopers
◇ = members selected by either Clean or Shifting Gapper
◆ = members selected by both Clean and Shifting Gapper

Indirect: based on modeling the PPS distribution of interlopers; interlopers are not rejected, mass estimates are based on all galaxies but with a weight that is proportional to the galaxy membership probability (e.g. *van der Marel et al. 2000*). There is a large variance in the surface density of interlopers around different clusters, making this technique not robust for individual clusters.

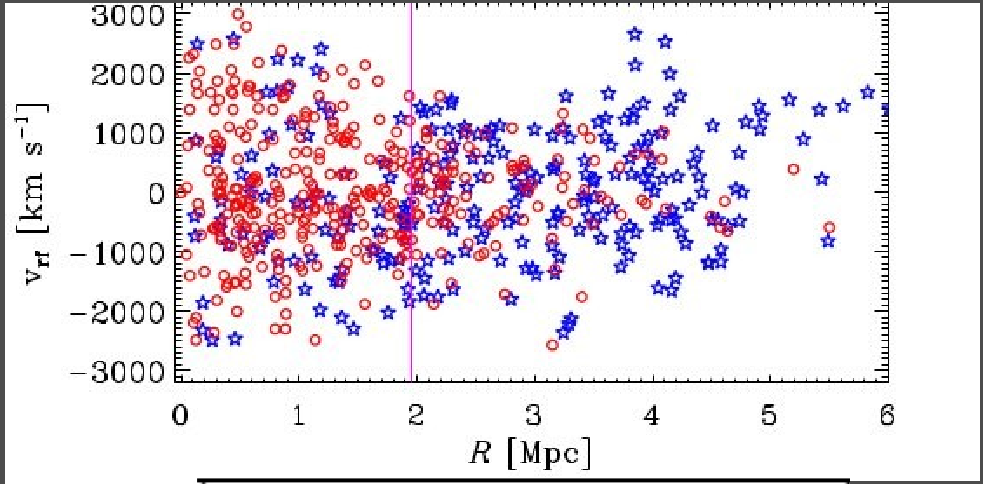


The surface density of interlopers in virial units for different clusters of a cosmological simulation (*Mamon, AB, Murante 2010*)

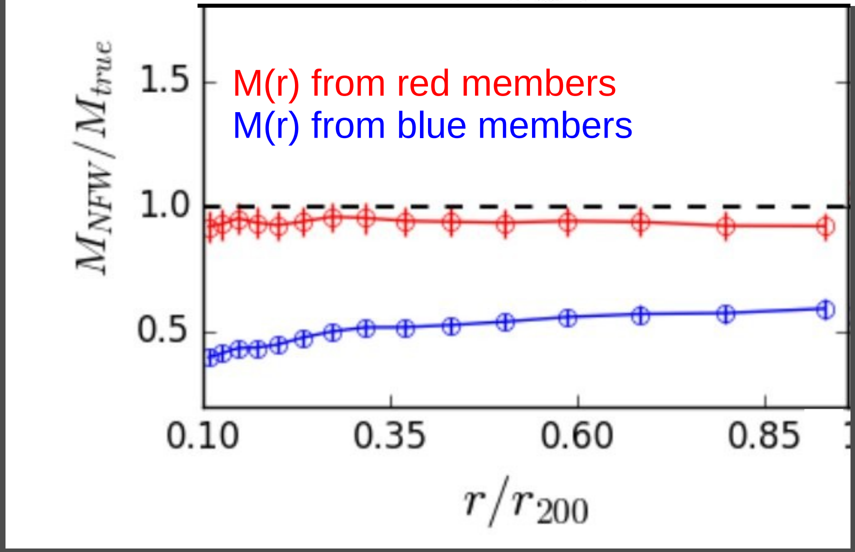
Masses & mass profiles

Galaxies

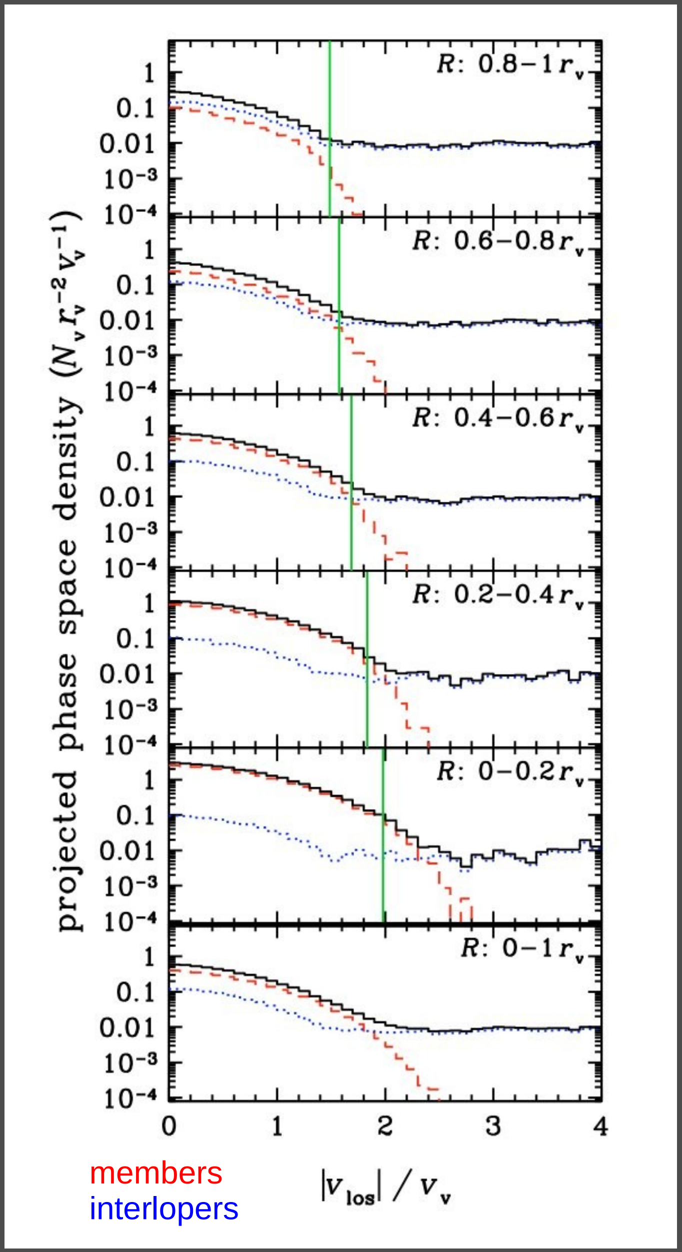
Since the fraction of interlopers increases with cluster-centric distance, and so does the fraction of blue/star-forming/spiral galaxies, it is easier to identify **red/passive/early-type cluster members**



Red and blue galaxies in a z=0.44 cluster *(AB et al. 2013)*



Tagliaferro, AB et al. (2021): MAMPOSSt solution for M(r) of clusters from numerical simulations using only red or only blue members identified in projected phase-space

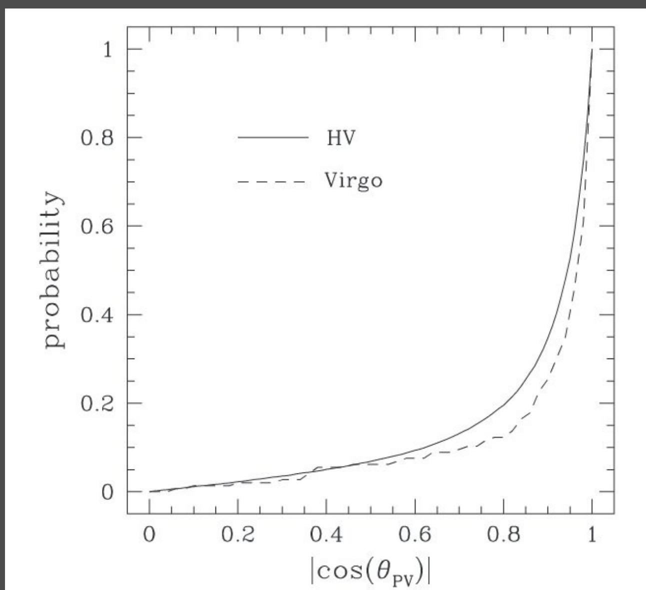


Mamon, AB, Murante (2010)

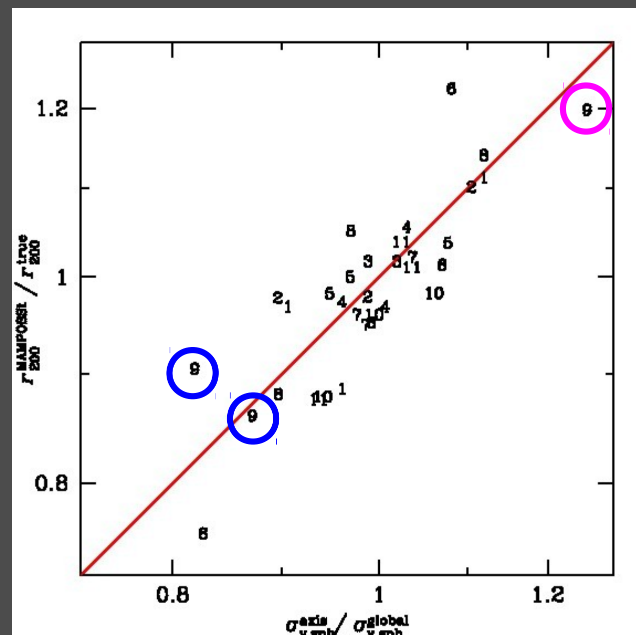
Masses & mass profiles

Galaxies

Clusters are not spherical. **Triaxiality** induces a systematic uncertainty in M(r) obtained from projected phase-space information

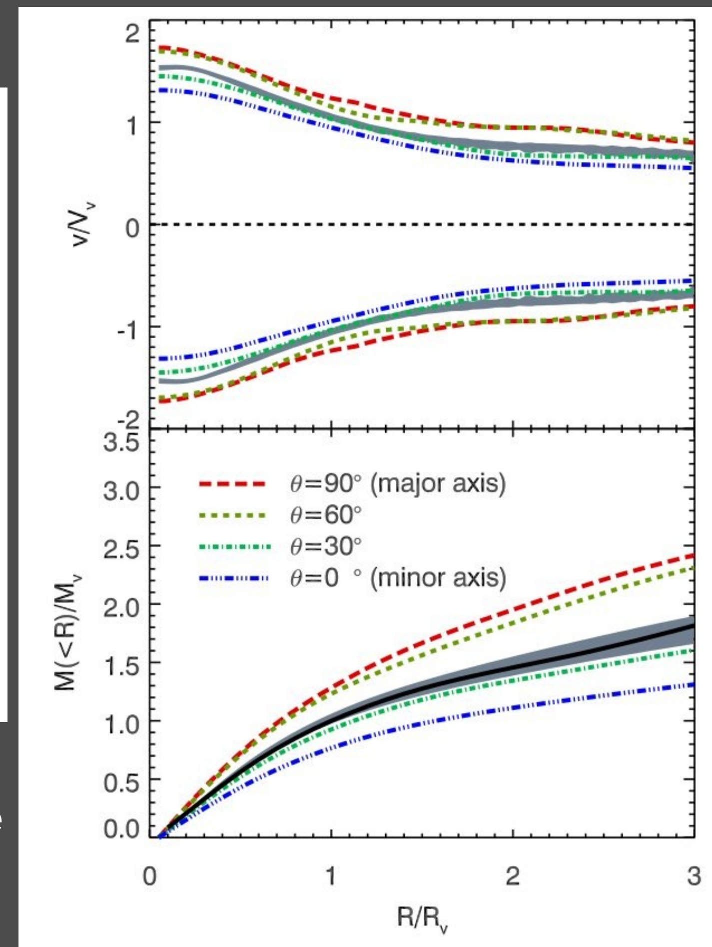


Position and velocity major axes of clusters from numerical simulations are aligned *(Kasun & Evrard 2005)*



Mamon, AB, Boué (2013): the ratio of estimated to true r_{200} correlates with the ratio of σ_{los} and global σ_v, larger for clusters with major axis along the los

(cluster "9": magenta indicate the result when the major axis is aligned with the los)

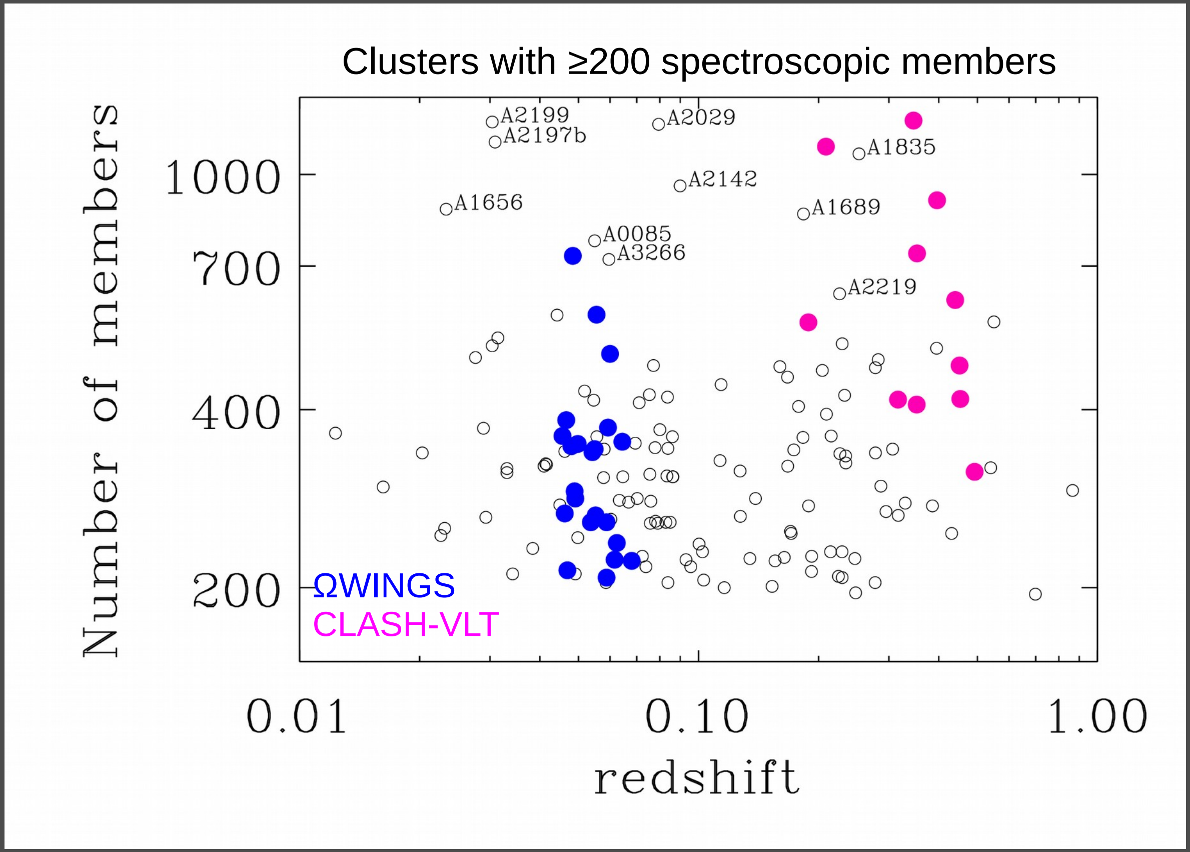


The Caustic mass estimate of a simulated cluster depends on the major axis orientation with respect to the los (*Svensmark et al. 2015*)

Masses & mass profiles

Galaxies

Poor statistics used to be the rule for spectroscopic samples of clusters before powerful multi-object spectrometers (e.g. VIMOS@VLT, AAOmega@AAT) and integral field unit spectrometers (e.g. MUSE@VLT) came into activity



Compilation by AB, last update Nov 2019.

For comparison, in the compilation of *Girardi, AB et al. (1993)* there were only 3 clusters with ≥ 200 spectroscopic members

Small statistics can still be an issue at **z~1** and for **poor clusters**

What can we do in these cases?

Masses & mass profiles

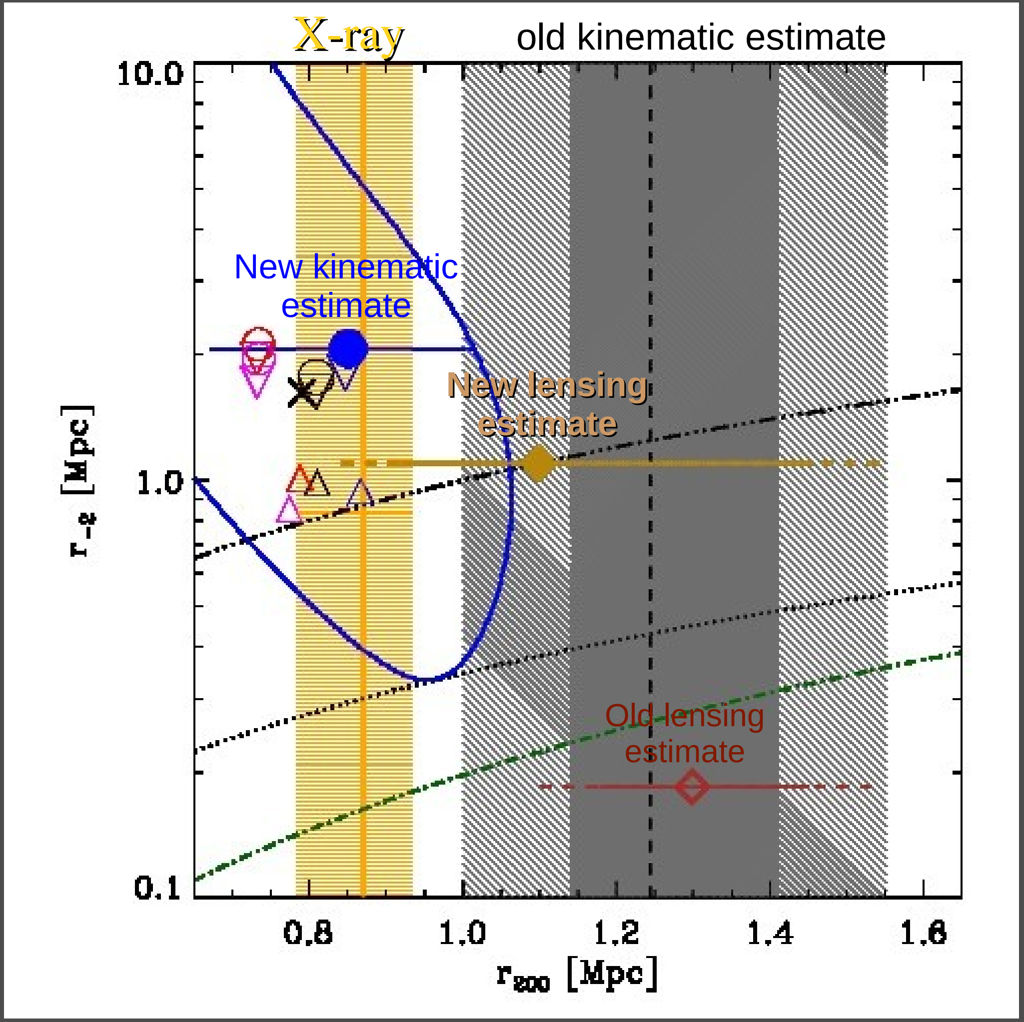
Galaxies

What we can do to alleviate the problem of poor statistics

1. Go back to the telescope and get more spectra

This typically means going fainter, implying much longer exposure time for the same number of objects, since $s/n \propto t_{exp}^{1/2}$ and since the fraction of interlopers among observed galaxies in the cluster field increases as one goes fainter

However, this generally pays off: an example from a z=0.17 cluster; large change in mass estimate by going from 20 to 200 spectroscopic members *(AB et al. 17)*



Masses & mass profiles

Galaxies

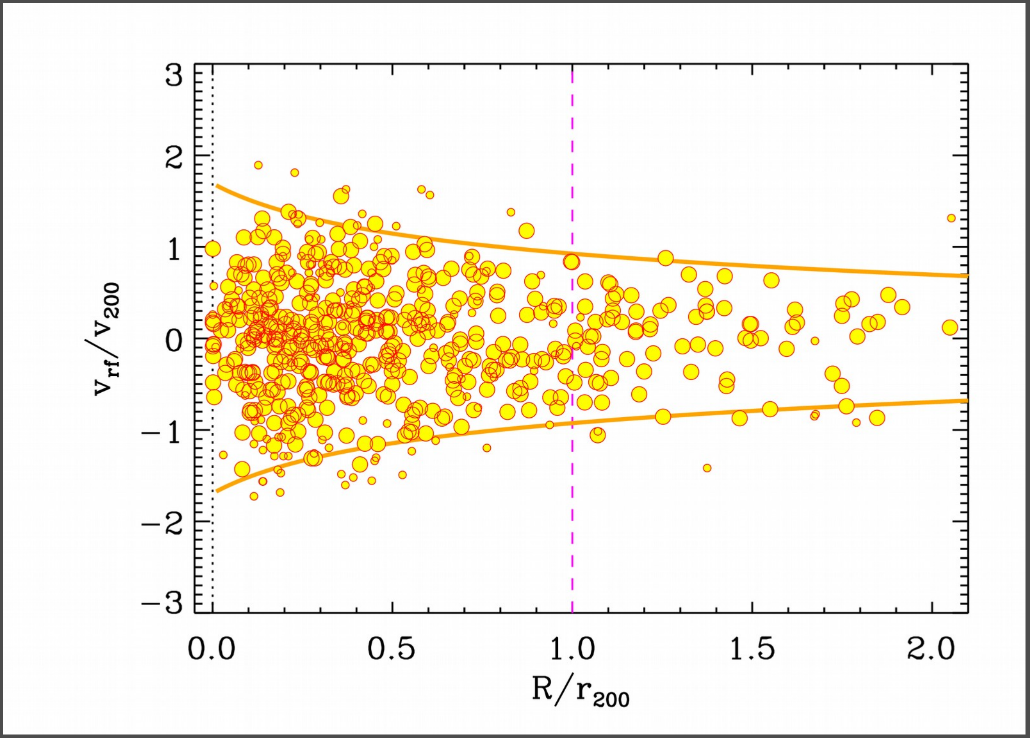
What we can do to alleviate the problem of poor statistics

2. Stack clusters

This must be done in such a way as not to mix inner virialized regions of massive clusters with outer unvirialized regions of low-mass clusters.

Since the concentration-mass relation is rather flat in the mass range of clusters, clusters are quasi-homologous in terms of their mass profiles, modulo the normalization term M_Δ.

Use $\mathbf{r_\Delta \equiv (2\ G\ M_\Delta / \Delta\ H_z^2)^{1/3}}$
to rescale galaxy cluster-centric distances: $R/r_{\Delta;}$
and $\mathbf{v_\Delta \equiv (2000\ G\ M_\Delta\ H_z / \Delta\ )^{1/3}}$
to rescale galaxy rest-frame velocities, v_{rf}/v_Δ



AB et al. (2021): stack of 14 clusters at 0.9 ≤ z ≤1.4 from the GOGREEN survey

The use of σ_{los} instead of v_Δ is also quite common (e.g. *Carlberg et al. 1997; AB & Girardi 2003*).
If the mass range is narrow enough, the stack can be done in physical units, without rescaling (e.g. *Rines et al. 2013*).

Masses & mass profiles

Galaxies

What we can do to alleviate the problem of poor statistics

3a. Make a better use of your data

Rather than trying to determine the mass of each and every cluster, use an independent mass proxy and calibrate it with all the information available.

An example with MAMPOSSt (*Capasso et al. 2019*):

$$\lambda = A_\lambda \left(\frac{M_{200c}}{M_{piv}}\right)^{B_\lambda} \left(\frac{1+z}{1+z_{piv}}\right)^{\gamma_\lambda}$$

1) Define a parametrized mass-richness relation

$$\mathcal{L}_i = \prod_{j \in gal} \mathcal{L}(R^j, v^j_{rf}, \lambda_i, z_i \mid \boldsymbol{p})$$

2) Sum all the cluster galaxy likelihoods

$$\mathcal{L} = \prod_{i \in clus} \mathcal{L}_i$$

3) Sum the cluster likelihoods ➔ MAMPOSSt probability of observing a galaxy ***j*** in a given position of the projected-phase-space of cluster **i**, given the set of parameters ***p***

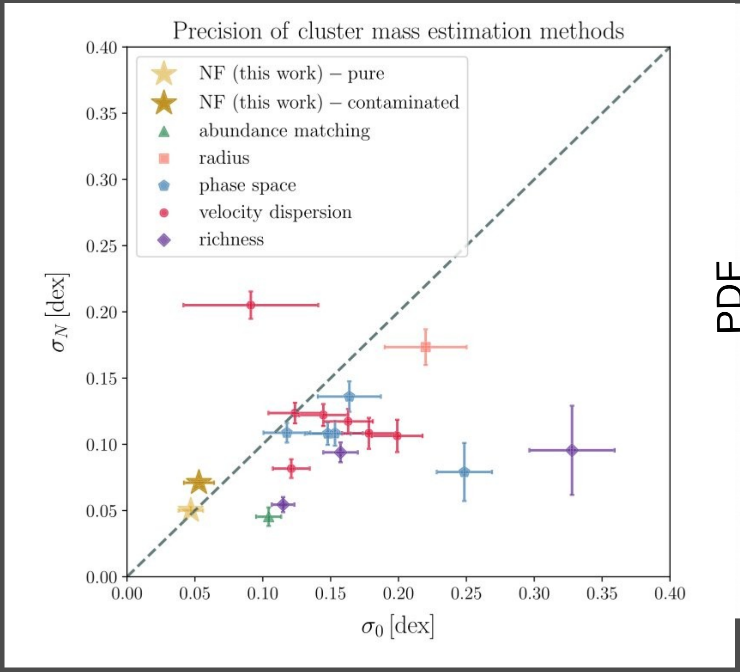
Wojtak et al. (2009) adopted this approach for the distribution function method.

Masses & mass profiles

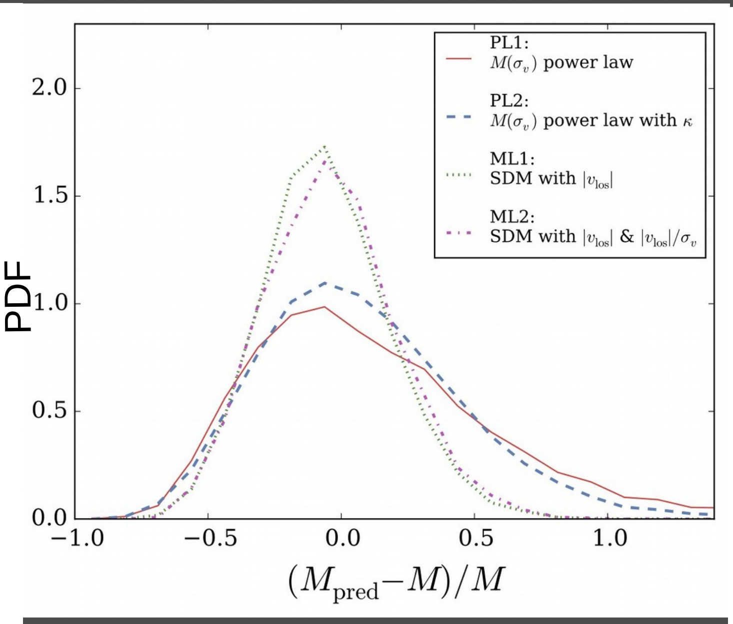
Galaxies

What we can do to alleviate the problem of poor statistics

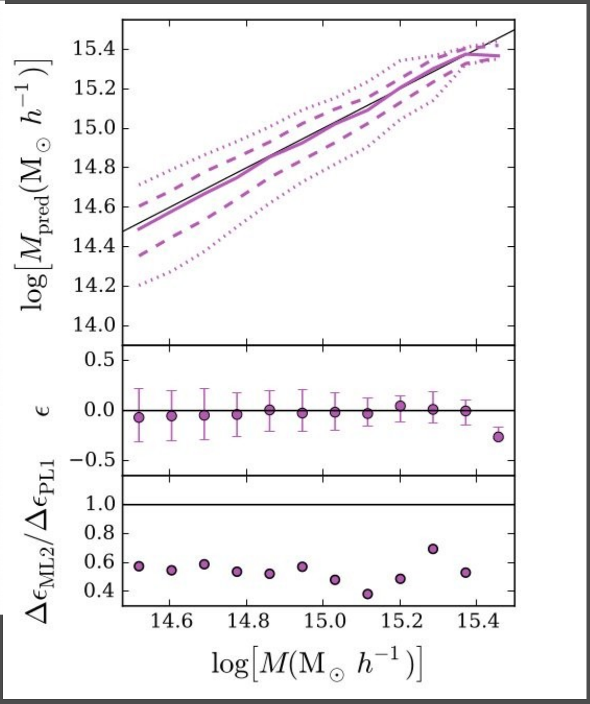
3b. Make a better use of your data: **Machine Learning**



Ramanah et al. (2020):
richness-dependent error σ_N vs. systematic error σ_0 for ML techniques (NF) and more traditional ones



Armitage et al. (2019):
Mass estimate with ML techniques compared to estimates from σ_{los} and σ_{los} + kurtosis



Ntampaka et al. (2015):
Mass estimate with ML techniques compared to estimates from σ_{los}

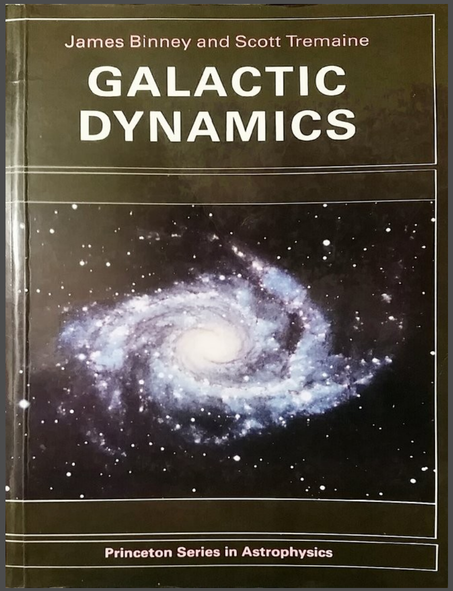
MAMPOSSt is still competitive with (or better than) ML with large (N~500) data-sets.
ML techniques do not yet address the mass profile determination.

Lecture 5 (part 1):

Masses & mass profiles

Based on:

Binney & Tremaine (1987), Chapters 4.1, 4.2, 4.3



Pratt et al. (2019), Sections 2.3, 2.5, 3

Published: 28 February 2019

The Galaxy Cluster Mass Scale and Its Impact on Cosmological Constraints from the Cluster Population

G. W. Pratt, M. Arnaud, A. Biviano, D. Eckert, S. Ettori, D. Nagai, N. Okabe & T. H. Reiprich

Space Science Reviews **215**, Article number: 25 (2019) | Cite this article

Kneib (2008):

J.-P. Kneib: *Gravitational Lensing by Clusters of Galaxies*, Lect. Notes Phys. **740**, 213–253 (2008)
DOI 10.1007/978-1-4020-6941-3_7 © Springer Science+Business Media B.V. 2008

Additional readings:

Girardi et al. (1998), ApJ, 505, 74 *(on the virial theorem)*
Mamon, AB, Boué (2013), MNRAS, 429, 3079 *(the MAMPOSSt method)*
Diaferio (1999), MNRAS, 309, 610 *(Caustic method)*