

Exoplanets

Timing and Astrometric methods

Planets and Astrobiology (2022-2023)

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Timing method

An orbiting planet generates a periodic oscillation of the position of the host star about the system barycenter

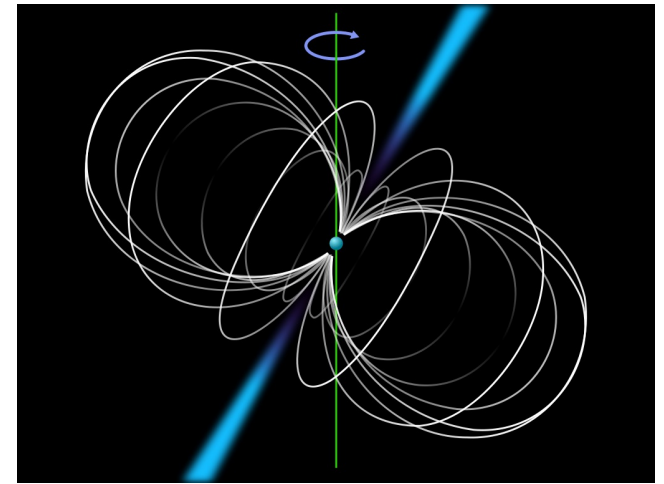
If the star also possesses regular periodic signatures of its own, then these can provide an alternative route to the dynamical detection of orbiting planets through the change in time of arrival of the pulses due to light travel time

The change of time of arrival is related to the displacement of the central star

$$\tau_p = \frac{1}{c} \frac{a \sin i M_p}{M_\star}$$

Several classes of astronomical objects offer this possibility:
radio pulsars, pulsating stars, and eclipsing binaries

Pulsars



End products of the stellar evolution of high-mass stars (~ 11 to $\sim 25 M_{\odot}$) that explode as a supernova, leaving a neutron star as a remnant of the collapse of their core

Neutron stars retain most of the angular momentum of their progenitors and attain high rotation speed because the progenitor's radius shrinks down to 10-15 km

They have very intense magnetic fields and emit a beam of radiation along their magnetic axis

The magnetic spin axis is generally disaligned with respect to the rotation axis and this misalignment causes the beam to be seen once for every rotation of the neutron star, which leads to the "pulsed" nature of its appearance

Pulsars timing method

- Based on the extreme regularity of the pulses emitted by pulsars in the radio band
 - Pulsar rotation periods are in the order of milliseconds
 - The methods consists in searching for variations in time of arrival (TOA) of the pulses, due to perturbations of the motion of the neutron star induced by an orbiting planet
 - For a planet with Earth mass in circular orbit with period $P \sim 1$ yr around a pulsar with mass $M = 1.35 M_{\odot}$, the time variation is ~ 1.2 ms (Wolszczan 1999)

Pulsars timing method

- **Derivation of the planet's mass**

- For a typical pulsar mass, $M_{\text{psr}}=1.35 M_{\odot}$, one can derive the following relation for the planet's mass, m_2 , its orbital period, P_b , and the semiamplitude of the TOA variations, Δt ,

Wolszczan (1999)
$$m_2 \sin i = 21.3 M_{\oplus} \left(\frac{\Delta t}{1 \text{ ms}} \right) \left(\frac{P_b}{1 \text{ day}} \right)^{-2/3}$$

- From this one derives a lower limit to the planet's mass since the inclination i is unknown

- **Advantages of the method**

- Thanks to the extreme precision of the measurements, it is possible to detect planets with terrestrial mass as well as multiple planetary systems

- **Disadvantage**

- Can only be used for neutron stars

Pulsars timing method

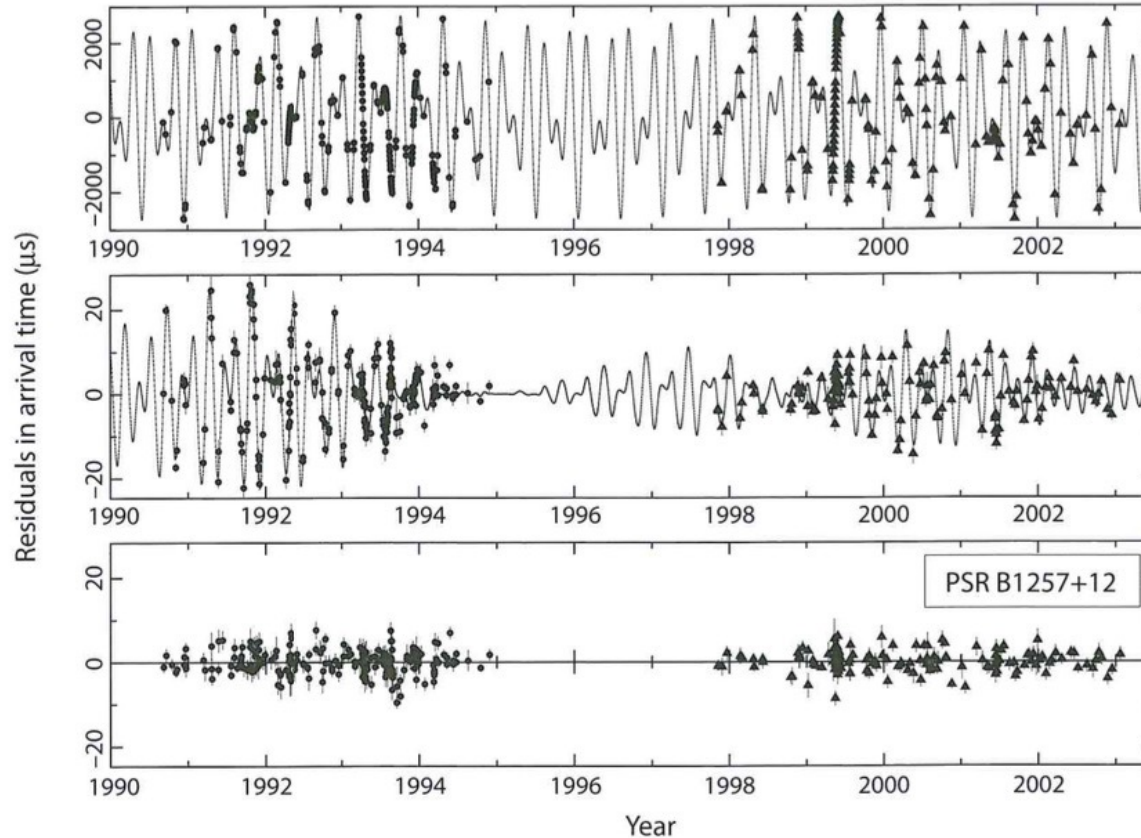
- First method that gave a firm detection of extrasolar planets
 - System with terrestrial-type planets detected before the first detection with the Doppler method
 - Wolszczan & Frail (1992)
 - Planets around a neutron star were unexpected
 - First of a series of discoveries that have shown that exoplanets can exist in a variety of situations, in many cases unexpected

Table 4.1: Orbital and physical parameters of the PSR B1257+12 planets. Planet designations follow those given in Wolszczan (1994a) and Konacki & Wolszczan (2003).

Parameter	Planet A	Planet B	Planet C
Projected semi-major axis (ms)	0.0030	1.3106	1.4134
Eccentricity, e	0.0	0.0186	0.0252
Orbital period, P (d)	25.262	66.5419	98.2114
Mass (M_{\oplus})	0.020	4.3	3.9
Inclination, i ($^{\circ}$)	–	53	47
Semi-major axis, a (AU)	0.19	0.36	0.46

Pulsars timing method

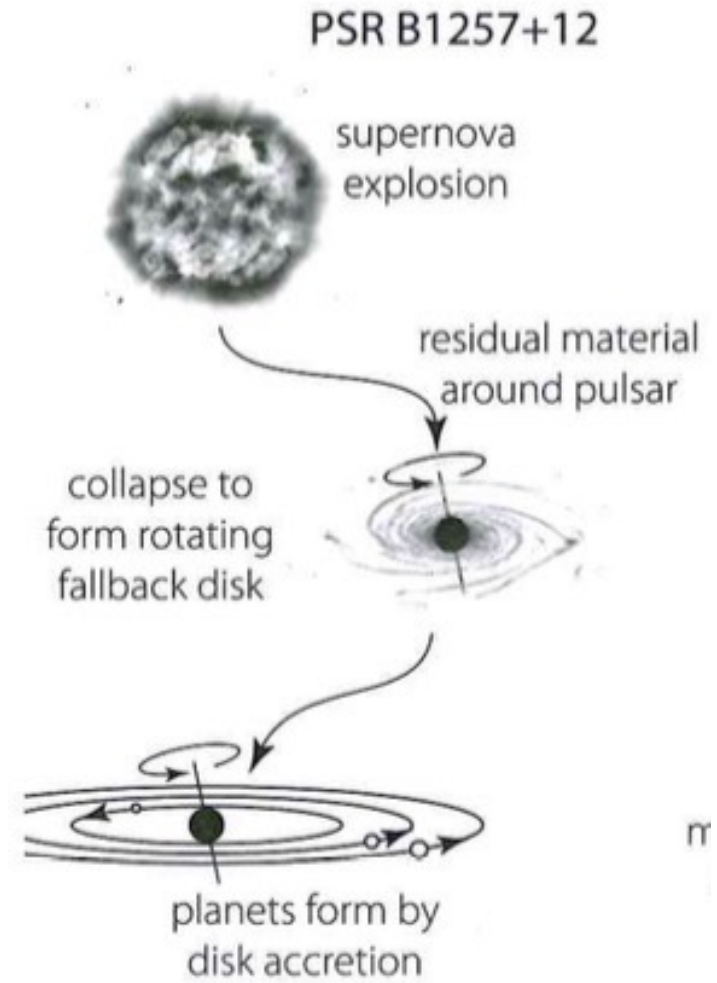
Time residuals for PSR B1257+12 observed at 430 MHz with the 305-m Arecibo telescope. Top: residuals for a model without planets. Middle: residuals for a Keplerian motion with three planets. Bottom: residuals after inclusion of planet-planet perturbations. (Konacki & Wolszczan 2003)



Planet survival after the supernova explosion

For PSR B1257+12 the planets are surmised to have formed from a “fallback disk” created after the supernova explosion

In other pulsars, different scenarios, featuring pre-existing main sequence planets, may take place



Pulsating white dwarfs

White dwarfs represent the end point of stellar evolution for stars up to $\sim 8 M_{\odot}$

They are common in the solar neighbourhood (more than ~ 100 within 20 pc)

As a white dwarf cools through certain temperature ranges, He and H in its photosphere progressively become partially ionized, driving multi-periodic pulsations with stable periods of hundreds to thousands of seconds

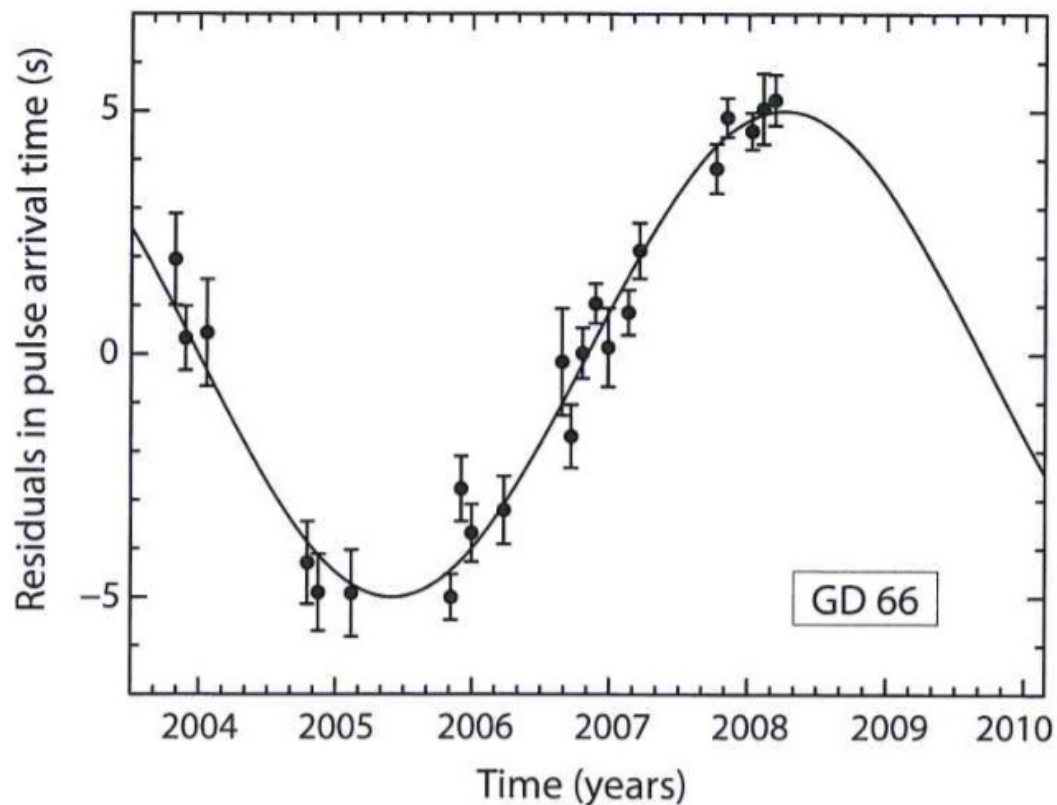
Whether planetary companions to white dwarfs exist, having survived the red giant branch and asymptotic giant branch stages of stellar evolution, will depend on the initial orbital separation, the stellar mass-loss rate and total mass loss, tidal forces and dynamical instabilities that may take place in the planetary system

Timing method

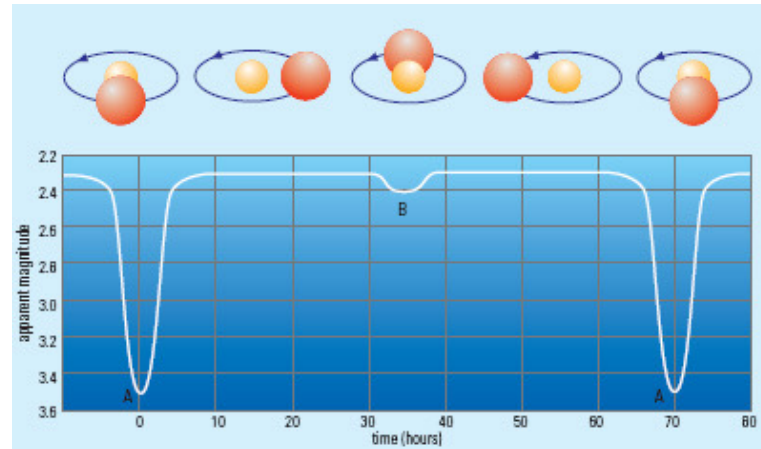
Application to pulsating stars: white dwarfs

Residuals in arrival time for one pulsation mode of the white dwarf GD 66

The timing residuals are consistent with a $2M_J$ planet in a 4.5 yr orbit



Eclipsing binaries



A planet orbiting both components of an eclipsing binary will result in periodically varying eclipse times as the binary circles the system barycenter

The majority of solar-type stars appear to reside in binary or multiple systems

Circumbinary planets can form and survive over long time scales in stable P-type orbits (see introductory lesson on exoplanets)

With timing accuracies of about 10 s for eclipsing binaries with sharp eclipses, one may detect circumbinary planets of $\sim 10 M_J$ in 10-20 yr orbits

Example of application to eclipsing binaries

HW Vir

Very short-period (2.8 h) eclipsing binary
Residuals of the eclipse timings measured since the early 1980s

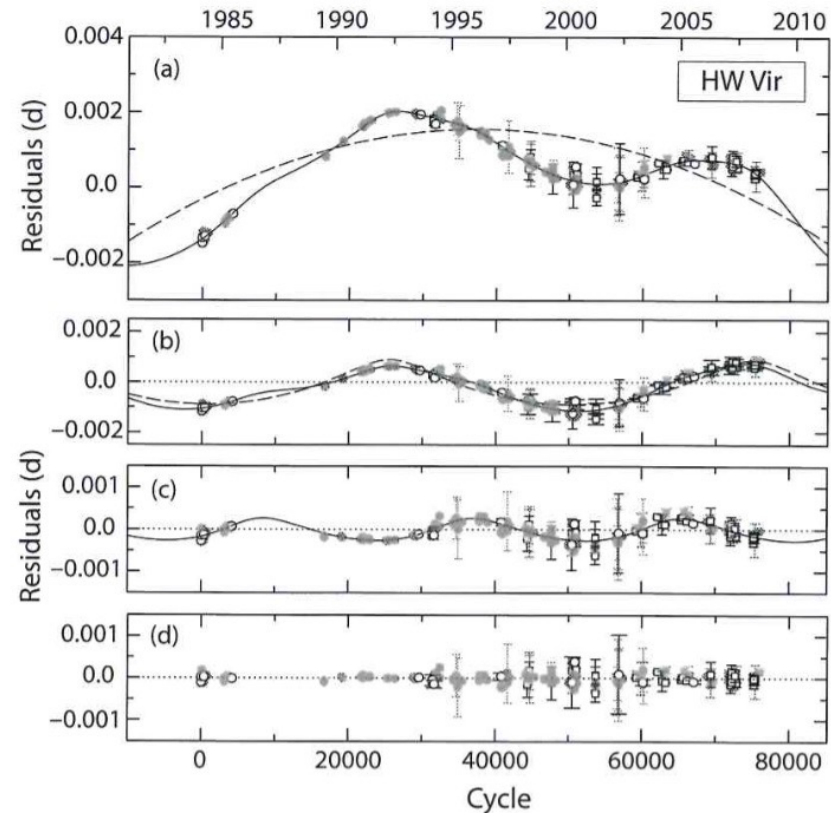
- (a) dashed curve: linear decrease of the rotation rate from stellar wind breaking
- (b) normalization of the residuals in (a)
- (c) residuals after including the effect of a 15.8 yr planet
- (d) residuals with respect to a two-planet model

Lee et al. (2009)

$$M_p \sin i = 19.2 M_J, P = 15.8 \text{ yr}, a = 5.3 \text{ AU}$$

$$M_p \sin i = 8.5 M_J, P = 9.1 \text{ yr}, a = 3.6 \text{ AU}$$

Timing method



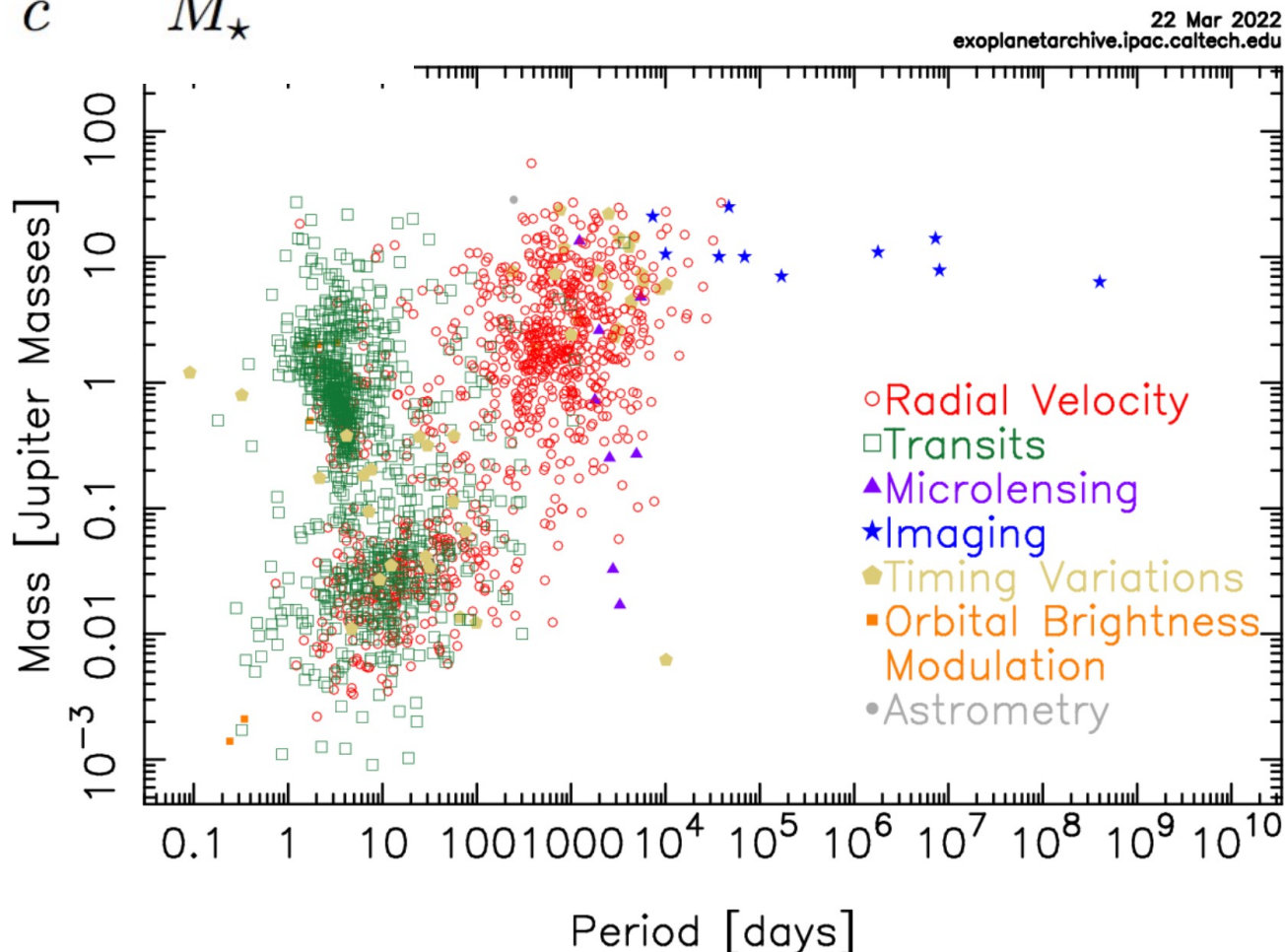
Timing method

currently (2022): 47 planets / 40 planetary systems / 5 multiple planet systems

Time variations tend to increase with increasing orbital period and planet mass

$$\tau_p = \frac{1}{c} \frac{a \sin i M_p}{M_\star}$$

Mass – Period Distribution



Astrometric method

Astrometric method

- **Astrometric measurements**
 - Variations of the proper motion of the host star around the barycenter of the planet-star system
 - The astrometric signal is composed of 3 terms:
 - 1) proper motion of the barycenter of the planetary system
 - 2) parallax effect (due to the Earth's orbital motion)
 - 3) reflex motion of the host star due to the planetary companion

The signal is larger when the orbit is seen face on ($\sin i \approx 0$)

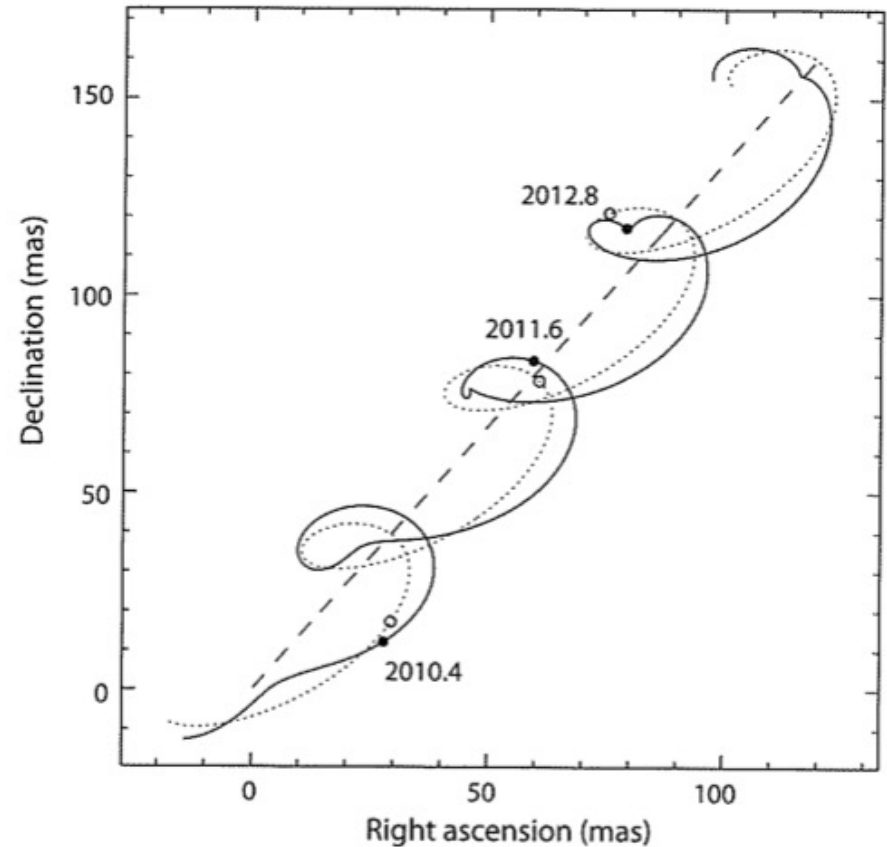


Figure 3.1: Schematic of the path on the sky of a star at $d = 50$ pc, with a proper motion of 50 mas yr^{-1} , and orbited by a planet of $M_p = 15 M_J$, $e = 0.2$, and $a = 0.6 \text{ AU}$. The straight dashed line shows the system's barycentric motion viewed from the solar system barycentre. The dotted line shows the effect of parallax (the Earth's orbital motion around the Sun, with a period of 1 year). The solid line shows the motion of the star as a result of the orbiting planet, the effect magnified by $\times 30$ for visibility. Labels indicate (arbitrary) times in years.

Astrometric method

- **Orbital and planetary parameters**

- The variations of the proper motion of the star due to the reflex motion scale as

$$\alpha \cong (M_p/M_*)(a/d)$$

a: semimajor axis; *d*: distance from Earth

- From reconstruction of the orbit one can obtain:
the period *P* and the orbital parameters *a* and *e*
and the inclination of the orbital plane, *i*
- From an estimate of the stellar mass *M*_{*} (combining observations with models of stellar structure) and of the distance *d*, one can obtain *M*_p

Astrometric method

- Size of the effect

$$\alpha = \left(\frac{M_p}{M_\star} \right) \left(\frac{a}{1 \text{ AU}} \right) \left(\frac{d}{1 \text{ pc}} \right)^{-1} \text{ arcsec}$$

- Examples of variations of proper motion expected for planets around a solar-type star located at a distance of 10 pc
 - 500 μas for a Jupiter-like planet
 - 0.33 μas for an Earth-like planet
- Measuring variations of this size poses a huge technological challenge
 - Typically, we can attain accuracies in the order of some milliarcsec for measurements of proper motions

Limit of the positional error

- Theoretical photon-noise limited positional error for a monolithic telescope:

$$\sigma_{ph} = \frac{\lambda}{4\pi D} \frac{1}{S/N}$$

where D is the telescope diameter, λ the wavelength and S/N the signal-to-noise ratio

For a $V=15$ mag, $\lambda = 600$ nm, $D = 10$ m, a system throughput of 40% and an integration time of 1 h, the resulting positional error due to photon noise alone is $\sigma_{ph}=30$ μ as

Even the best astrometric accuracies fall far short of this theoretical performance, mainly as a result of atmospheric turbulence and differential chromatic refraction

Astrometric method

- Potential advantages
 - It is effective for planets in wide orbits
 - It is virtually unaffected by stellar variability and activity effects
- Potential problems for the detection of terrestrial planets
 - The signal expected for terrestrial-type planets is well below the theoretical limits of positional error
 - In a multiple system with planets of different sizes the astrometric signal would be dominated by the reflex motion induced by giant planets

Astrometric method

Favours the detection of long-period, giant planets in nearby, low mass stars

So far, there are some detections,
but generally above the planet mass threshold

Showing 15 planets / 4 planetary systems / 0 multiple planet systems

Show entries

Planet Search

Planet	Mass (M_{Jup})	Radius (R_{Jup})	Period (day)	a (AU)	e	i (deg)	Ang. dist. (arcsec)	Discovery
WISE J0458+6434 A	57	—	15500	1	0.18	76.5	—	2019
DENIS-P J082303.1-491201 b	28.5	—	246.36	0.36	0.345	56.6	—	2013
DE0630-18 (bc)	53	—	—	—	—	—	—	2020
DE0630-18 a	58	—	—	—	—	—	—	2008
TVLM 513-46546 b	0.38	—	220	0.3	0	71	—	2020
SDSS J080531+481233 B	66.28	—	740.43	0.82	0.423	111.85	—	2016
2M1059-21 b	66.95	—	690	0.8	0.146	32.9	—	2020
WISE J0458+6434 B	14	—	15500	4	0.18	76.5	—	2019
2MASS J15344984-2952274 A	30.1	—	—	—	—	—	—	2008
2MASS J15344984-2952274 B	28.2	—	5550	2.3	0.24	84.3	—	2008
SDSS J080531+481233 A	60	—	—	—	—	—	—	2016
2MASS J0249-0557A	48	—	—	—	—	—	—	2018
2MASS J0249-0557 (AB) c	11.6	—	—	—	—	—	—	2018
2MASS J0249-0557B	44	—	—	—	—	—	—	2018
HD 176051 b	1.5	—	1016	1.76	0	—	0.117412	2010

Astrometric method

- **Results in combination with other observational methods**
 - The combination of astrometric measurements with measurements obtained with other methods, such as the Doppler method, may provide significant observational constraints

- **Example**

- Astrometric signal of a long period, nearby planet:

- ϵ Eri b ($d=3\text{pc}$)

- $\alpha = 1.88 \text{ mas}$

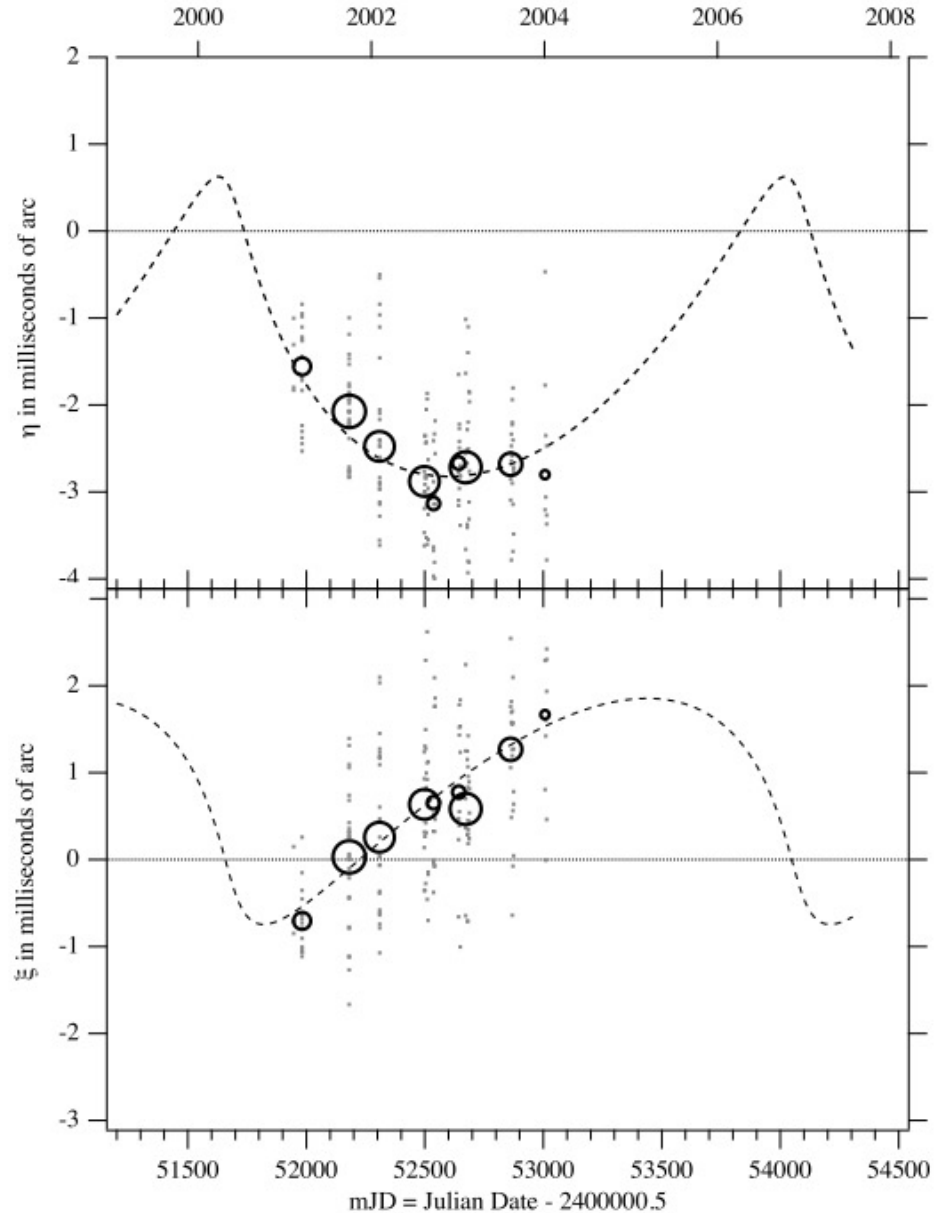
- $M_p = 1.55 M_J$

- $P = 6.85 \text{ yr}$

- $a = 3.39 \text{ AU}$

- [Benedict et al. 2006](#)

The result has been obtained by combining the radial-velocity data with astrometric data from Earth and from space (HST)



Example of a simultaneous fit of Doppler and astrometric data
The planet's mass can be significantly constrained combining both types of data
GJ 317b (Anglada-Escudè et al. 2012)

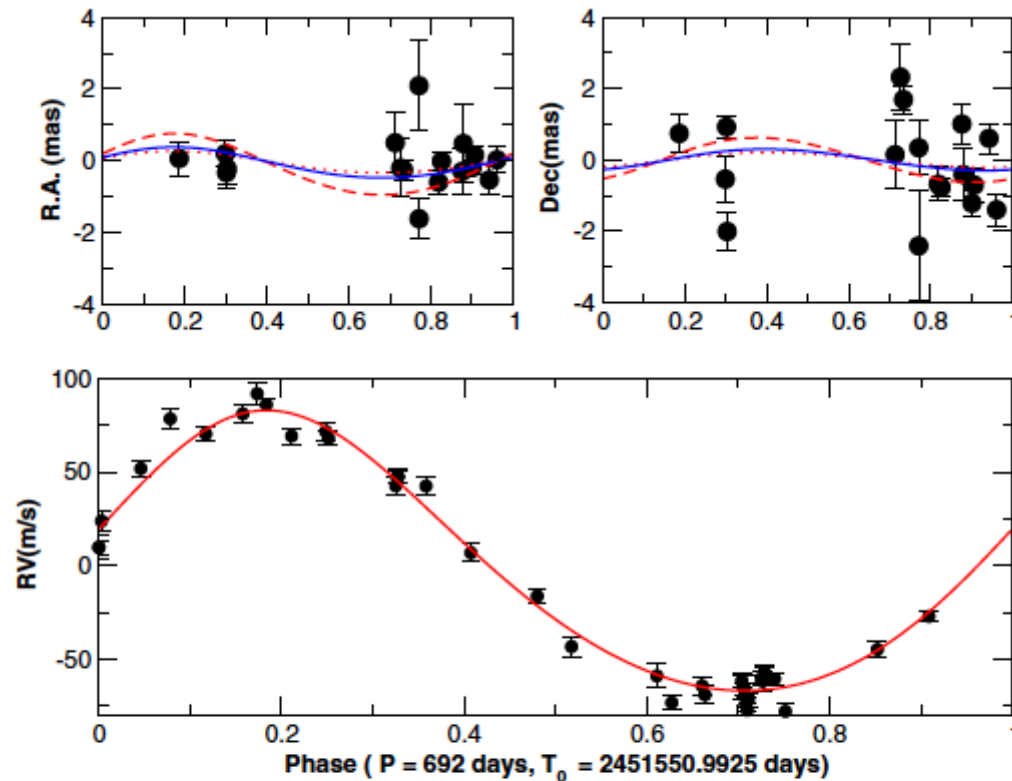


Figure 5. Best-fit astrometry + RV joint solution for GJ 317b. Top panels are R.A. and decl. as a function of time. The dashed red line corresponds to the signal of a $4.0 M_{\text{Jup}}$ mass planet. The dotted red line corresponds to the signal of a planet with the minimum mass allowed by the radial velocities ($1.8 M_{\text{Jup}}$). The best-fit solution is plotted as a blue line. Bottom panel: the radial velocity data plotted with the best-fit solution after removing the signal of planet c.

Astrometric methods: prospects for the future

- Gaia mission (ESA)

- <http://gaia.esa.int>

- Unbiased survey (without pre-selection) of 10^9 point-like Galactic sources (6-20 mag)
 - Astrometric precision, up to 10 μas for the brightest sources
 - Intermediate resolution spectra for $\sim 150 \times 10^6$ stars

- Astrometric results expected in DR3 (data release 3)

- Expected astrometric detections of exoplanets in the vicinity of the Sun

- Examples of expected accuracy:

- 6 μas (at V=6 mag) and 200 μas (at V=20 mag) for a G2V-type star

Exoplanet research will also benefit from Gaia mission in an indirect way:

Significant improvement in the calibration of stellar physical properties

➔ Accurate determination of stellar radii and masses

➔ As a result, accurate determination of exoplanet radii and masses through indirect methods